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GEOTECHNICAL INSTITUTE
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A Report on Half Scale Tests

*By* ARNE RINKERT

## Measurements in Grain Silos

during Filling and Emptying

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Ivar Hæggströms Boktryckeri AB
Stockholm 1959

# Measurements of the Pressures of Filling Materials against Walls

## Preface

During the course of years the Swedish Geotechnical Institute has dealt with various cases of measurements of earth pressures. The process often involves great difficulties, and experiments on large structures are also very expensive. It would, therefore, appear to be of value to report case records obtained.

A nnmber of tests and measurements on an abutment on half scale were performed by the Stockholm Harbour Board and supervised by the Institute. The report is entitled "Earth Pressure from Friction Soils. A Report on Half Scale Tests", by Arne Rinkert, constituting the first part of the present publication (p. 3 to 46).

Since case records on silos are relatively rare in geotechnical literature the Institute has decided to publish the results of certain full scale tests in its Proceedings series. The report is entitled "Measurements in Grain Silos during Emptying and Filling", by Werner Bergau (p. 47 to 70).

For practical reason, the two reports are issued in one publication under a common title, *viz.*, "Measurements of the Pressures of Filling Materials against Walls".

Stockholm, October, 1959

SWEDISH GEOTECHNICAL INSTITUTE

# Earth Pressure from Friction Soils

**A Report on Half Scale Tests**

**by *Arne Rinkert***

## Preface

The tests on earth pressure against retaining walls described in this report were financed by the Swedish State Committee for Building Research, the Public Works of Stockholm, Street Department, and the Stockholm Harbour Board. The Building Department of this latter institution was responsible for conducting the tests. The Swedish Geotechnical Institute approved the devices used and followed the test procedure.

At the Harbour Building Department the tests were planned and supervised by Mr Herman Jansson, Chief Engineer, Mr Arvid Wickert, Head of the Design Department, and Mr S. Kasarnowsky, Departmental Engineer. Mr A. Rinkert, civil engineer, was responsible for management of the tests and prepared this report.

The report was later examined and to some extent supplemented by the undersigned Institute. The Stockholm Harbour Board made a grant towards the cost of publication.

Stockholm, January, 1959

SWEDISH GEOTECHNICAL INSTITUTE

## Contents

## 1. Synopsis

In Sweden earth pressure from friction soils against retaining walls and abutments was up to the beginning of the 1940s calculated in accordance with the so-called classical earth pressure theory. However, as early as in the 1920s tests indicated that the earth pressure against a non-yielding wall, called earth pressure at rest, could be far greater than the active earth pressure. These and later experiments had shown that the intensity of the earth pressure at rest could be up to 0.4-0.5 of the vertical pressure whereas, at an angle of friction of 32°, the active earth pressure amounts, as known, to only 0.3 times the vertical pressure. With an angle of friction of 42° the corresponding figure is 0.2.

About 1940 an increase of the dimensioning earth pressure to the at rest value was actualized in Sweden. As this however would result in considerable expense, it was found necessary to make a more detailed examination in respect of earth pressure against retaining walls.

Research funds for an investigation were placed at the disposal of the Swedish State Committee for Building Research, the Public Works of Stockholm, Street Department, and the Stockholm Harbour Board. The Building Department of the latter institution was entrusted with the task of carrying out the investigations, while the Swedish Geotechnical Institute was to supervise the work.

The tests were carried out on a reinforced concrete wall 2 metres high and 6 metres long. The thickness was 0.2 m. The wall was mounted on two bearings laid in line with the surface of the wall facing the filling and at a third point located one metre in front of the bearings. This latter point was arranged by providing the wall with a sturdy cantilever footing. The wall was free to travel between two non-yielding side walls of reinforced concrete coated with sheet-metal and greased to reduce the friction between the filling and the side walls. The relatively great height to length had been chosen for the same reasons, *viz.*, that the side-wall friction should have only a slight effect on the earth pressure against the wall. The movement of the wall caused by earth pressure could be controlled.

To simplify the measurements, only the overturning moment acting on the wall was measured. When treating the results of the tests, the moment had been converted into a non-dimensional coefficient $K$ in the expression of the horizontal component of the earth pressure resultant $E = K \cdot \gamma h^2/2$, where $\gamma$ is the unit weight of the filling and $h$ the height of the wall. In all tests the upper surface of the filling was horizontal and flush with the top of the wall.

Two kinds of filling material were investigated, *viz.*, *macadam* and *pebbles*.

The *macadam* had a size of 32—64 mm, a unit weight of 1.34 t/m$^3$ and an angle of friction of 40°. A $K$-value of 0.34 was obtained with a non-yielding wall whereas, with a wall movement equal to about 1/3000 of the wall height, the pressure had decreased to a $K$-value of 0.19, or about the same as for the active earth pressure of the material.

Tests with an overload on the macadam filling were carried out at a given wall yielding. Its average movement due to earth pressure from the *back-filling* alone amounted to about 1/5000 of the wall height. On the assumption that the earth pressure caused by the overload was uniformly distributed along the wall and of an intensity equal to $K$ times the overload, it was found that the $K$-value was about the same as for the earth pressure caused by filling alone at the same wall yielding.

The *pebbles* consisted of round and oval stones of size 16-32 mm. The unit weight was 1.58 t/m$^3$ and the angle of friction 40°. With a non-yielding wall the $K$-value at rest was 0.30 and, with an average wall movement equal to about 1/800 of the wall height, a $K$-value of 0.19 was obtained. This is about equal to the value for active earth pressure.

The tests thus showed that there exists a kind of higher value for the earth pressure, a value at rest, which is larger than the active pressure. However, in the case of the materials tested, it was appreciably lower than values disclosed by earlier investigations on sand. There may have been several reasons for this, *e.g.*, different degrees of compaction, differences in the Poisson's ratio and the effect of friction along the front wall.

If—as is usual when calculating the stability of retaining walls—the generally favourable influence of the friction along the wall is disregarded, it should also justify a reduction of the horizontal component for the earth pressure caused by the friction.

The tests also showed that the wall movement required to reduce the earth pressure to the active level is, at least as far as macadam is concerned, comparatively slight and probably occurs in many existent abutments and retaining walls. It would therefore seem justified only to use the higher pressure disclosed by the tests as the basis when calculating retaining walls in such cases where it can he assumed that the travelling capacity of the construction is negligible.

## 2. Introduction

Up to the beginning of the 1940s the usual practice in Sweden was to calculate the earth pressure from friction soils against retaining walls and similar constructions in accordance with the classical theory for earth pressure. As is known, this theory assumes that the shear strength of the soil at an arbitrary point on a slip surface is proportional to the normal pressure and that the material has no tensile strength.

The magnitude of the pressure against a wall is calculated on the assumption that a wedge-shaped portion of the earth nearest the wall slides out along a slip surface, which means that the friction along this surface is fully mobilized. In order that this sliding can take place the wall must as a rule, when subject to active earth pressure, make a certain movement away from the earth mass and, in the case of passive earth pressure, towards the earth mass.

In the case of a vertical wall, and a horizontal upper surface of the filling, and disregarding the friction between the wall and the filling, the active earth pressure is

$$E_a = \frac{h^2\gamma}{2} \cdot \mathrm{tg}^2\left(45^\circ - \frac{\varphi}{2}\right)$$

where $\gamma =$ unit weight of the filling
$\varphi =$ angle of internal friction
$h =$ height of the wall

The passive earth pressure is calculated on the same assumptions to

$$E_p = \frac{\gamma h^2}{2} \cdot \mathrm{tg}^2\left(45^\circ + \frac{\varphi}{2}\right)$$

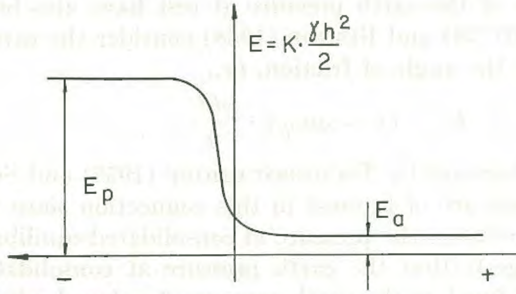


*Fig. 1. Earth pressure as a function of the movement of the wall, against filling (–) and from filling (+), in principle.*

For material with an angle of friction of 32° we have

$$E_a = 0.31 \cdot \frac{\gamma h^2}{2} \text{ and } E_p = 3.25 \cdot \frac{\gamma h^2}{2}$$

In this case the passive earth pressure is thus over ten times larger than the active.

However, the classical earth pressure theory says nothing about the magnitude of the movements of the wall required to bring about active and passive earth pressures. Neither is any mention made of the magnitude of the earth pressure in the case of movements smaller than those required to reach the limit values in these two cases, *cf.* Fig. 1.

An item of especial interest is the magnitude of the earth pressure in the case of non-yielding walls. Many constructions are so rigid that the friction of the earth material cannot be fully mobilized. Tests have shown that the earth pressure at zero wall movement is larger than the active earth pressure. This pressure has been called earth pressure at rest.

If the earth is regarded as an isotropic, elastic body which follows Hooke's law, the magnitude of the earth pressure at rest can, theoretically (*cf. e.g.* TSCHEBOTARIOFF, 1951) with the condition of no lateral movement and a vertical wall, be calculated as being

$$E_o = \frac{1}{m-1} \cdot \frac{h^2\gamma}{2}$$

in which $\frac{1}{m}$ stands for Poisson's ratio. The pressure would thus be independent of the internal friction of the material. However, the implication of Poisson's ratio for different types of earth is little known and seems to have no constant value; furthermore, it is difficult to establish experimentally (JAKOBSON, 1957).

If the stress ellipsoid of the adjoining earth mass is oblique in relation to the wall, *i.e.*, there are shearing stresses, the earth pressure at rest can have a different value from that indicated above.

Other definitions of the earth pressure at rest have also been suggested. For example, JÁKY (1937/38) and BISHOP (1958) consider the earth pressure at rest to be a function of the angle of friction, *i.e.*,

$$E_o = (1 - \sin \varphi) \cdot \frac{\gamma h^2}{2}$$

Suggestions pnt forward by TSCHEBOTARIOFF (1957) and SCHMID (1957) concerning cohesion soils are of interest in this connection since they contend that there is a relation between the pressure at consolidated equilibrium and plasticity index. Schmid suggests that the earth pressure at consolidated equilibrium in cohesion soils be defined as the earth pressure developed when the time-rate of the strain is zero, *i.e.*,

$$E = E_{ce} \text{ when } \frac{\partial \varepsilon_{ij}}{\partial t} = 0$$

In this case, $\varepsilon_{ij}$ is the total strain at point $i$ in the arbitrary direction $j$.

In the same volume BISHOP (1957) protests the definition used by Schmid aud claims that it cannot give the same results as the classical definition that the earth pressure at rest is the earth pressure at no lateral strain. In a later work (BISHOP, 1958), he gives certain test results for the ratio $E_a/E_o$ which agree closely with the theoretically calculated values, if $E_o$ is assnmed to follow Jáky's expression.

OSTERMAN (1958) advances the opinion that, as regards the pressure from friction soil against a non-yielding wall, the values may be between the active and the passive and, in exceptional cases, even beyond these limits. However, it should be possible to assume that especially high valnes would change very quickly in the event of a movement of the wall.

As mentioned before it is only in recent years that it has been possible to determine some approximations of the ratio $\frac{1}{m}$ (KJELLMAN and JAKOBSON, 1955; JAKOBSON, 1957) in the above formula and, in this way, to get an indication of the magnitude of the earth pressure at rest. On the other hand, some tests involving direct measurement of the earth pressure against a non-yielding wall have been carried out. The best known of these investigations are probably the works of TERZAGHI (1920) who made a number of tests, although only on a small scale. The test wall was about 10 cm high, and the earth pressure at rest was found to be $0.42 \cdot \gamma \cdot h^2/2$. New comprehensive tests, on a much larger scale, were later carried out by TERZAGHI (1930, 1934) and the earth pressure at rest for the material investigated (sand) was then found to be $0.405 \cdot \gamma \cdot h^2/2$. The height of the wall was about 1.5 metres and the mean movement of the wall at active earth pressnre was abont 1/3000 of the wall height.

KJELLMAN (1936) studying the deformation properties of certain soils with cubes of dimensions 62 × 62 × 62 mm found an at rest coefficient of about 0.5.

According to tests made in 1948 by GRADOR, the coefficient of the earth pressure at rest should first reach the value of 0.45 when the material was densely compacted. With loose material he arrived at a value of 0.29. However, it should be pointed out that the coefficients of the earth pressure at rest for different states of compaction are of little interest unless this state is accurately defined in one way or another. At least it is theoretically possible to compact to such an extent that passive earth pressure is obtained.

Grador's experiments indicate that active earth pressure requires large wall movements—about 1/70 of the wall height. This does not agree very well with other experiments but, as far as can be judged from the report, this may partially be due to the non-elimination of the friction along the side walls. This causes too low an "active" earth pressure if this latter is defined as the earth pressure at large wall movements. As a result, the displacement of the wall required to obtain "active" earth pressure will be greater. The necessary movement also increases with the initial degree of compaction. Since the earth pressure at rest is measured at zero wall movement, the friction along the side walls cannot, however, have had any effect, and consequently Grador's value for the earth pressure at rest may be correct.

In the beginning of the 1940s, the Board of Roads and Waterways of Sweden prescribed that the earth pressure at rest should, in certaiu cases, be used as the yardstick for the dimensioning and stability calculations of retaining walls and similar constructions. The economic consequences of this requirement would however be considerable. In the case of rock waste with an internal angle of friction of 42°, the active earth pressure is 0.2 times the vertical pressure. Since the earth pressure at rest was considered to be 0.45 times the vertical pressure, the new requirements meant an increase in the earth pressure by 2.3 times. Soil with an internal angle of friction of 32° has an active earth pressure of 0.3 times the vertical pressure, and consequently the changeover to earth pressure at rest should result in a 1.5 times increase in the earth pressure. However, to cut down the economic consequences, the safety margins were somewhat reduced simultaneously.

There were different opinions among technicians concerning the jnstification of calculating retaining walls and abutments on the basis of earth pressure at rest. As a result, in 1945 the STATE COMMITTEE FOR BUILDING RESEARCH in Sweden called for a conference on "Earth Pressure at Rest in Connection with Earth Pressure Calculations". At this conference the "earth pressure at rest advocates" were mainly represented by the Swedish Geotechnical Institute, the tests made by Kjellman and Terzaghi (referred to above) forming the technical basis. It was also stated that some abutments had been subject to movement and that the reason was considered to be that too low values of earth pressure had been used in the calculations. Opponents of the earth pressure at rest theory considered that the values had for the best part been determined by laboratory tests under conditions that seldom occurred in practice. Thus, as a rule, it is seldom that, in practice, a wall lacks the possibility to move, it was said. In the case of highway embankments, too, there is always the possibility of movement, at least sideways. The fact that abutments had travelled was not, as had been considered, to be attributed to too low an earth pressure but to other causes.

It was found at the conference that further investigations and experiments would be necessary to establish the true facts of the matter under discussion.

The question was of especial interest to the Stockholm Harbour Board in view of the plans to construct a high bridge at Skanstull in Stockholm. The Board therefore considered it a matter of importance to initiate tests to decide whether earth pressure at rest really must be allowed for.

The Board of Roads and Waterways of Sweden later modified its requirements concerning the calculation of earth pressure against retaining walls. It is thus stated in the Board's "Design Standards 1947" that retaining walls and abutments are to be calculated for *active* earth pressure. In cases where the construction may be subject to vibrations caused by passing traffic, the horizontal component of the earth pressure—in cases of normal load—is to be increased by 25 %. For walls and abutments founded on piles or on rock the earth pressure is also to be increased by 25 %—thus making the total increase 50 %—but such a case may be considered as exceptional, thus with allowance of especially high stresses or low safety.

After the interested parties had discussed the question with the State Committee for Building Research, the Committee voted funds for an investigation to be carried out under the auspices of the Stockholm Harbour Board, which together with the Public Works of Stockholm, Street Department, also contributed the additional funds. The tests were to be supervised by the Swedish Geotechnical Institute.

The investigation commenced in 1946 and, as regards the first stages—dealing with earth pressure from macadam and from overloads on macadam against a retaining wall of normal yielding—a report was presented at the "Second International Conference on Soil Mechanics and Foundation Engineering" in Rotterdam (JANSSON, WICKERT and RINKERT, 1948).

Later on, the test device was made more complete so that the yield of the wall could be varied, and a new series of tests in respect of the earth pressure as a function of the wall movement was carried out. In addition to macadam, the use of pebbles as a back filling material was also investigated.

## 3. Test Arrangements

### 3 a. General Description

The test wall and the test arrangements are shown in Figs. 2 a, 2 b, 3 and 4. The wall was 2 metres high and 6 metres long. The reason for the relatively great length was the desire to reduce the effect of friction along the side walls on the earth pressure against the wall. The height/length ratio in question resulted in the friction effect being small, and to reduce the wall friction further, the side walls were faced with a 0.9 mm metal sheathing coated with graphite and grease.

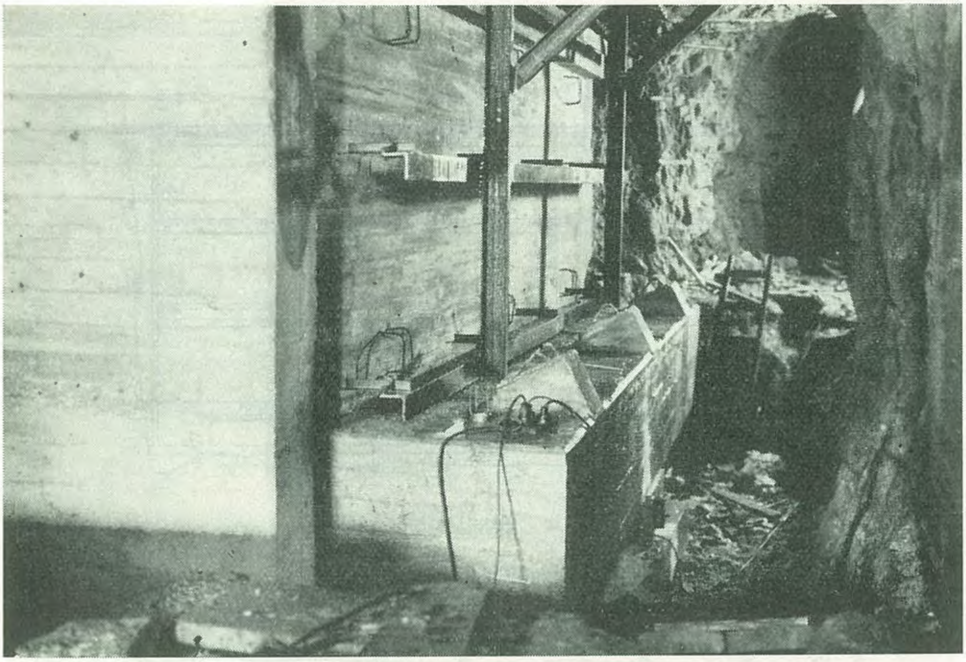

*Fig. 2 a. Photo of test wall before installation of gauges.*

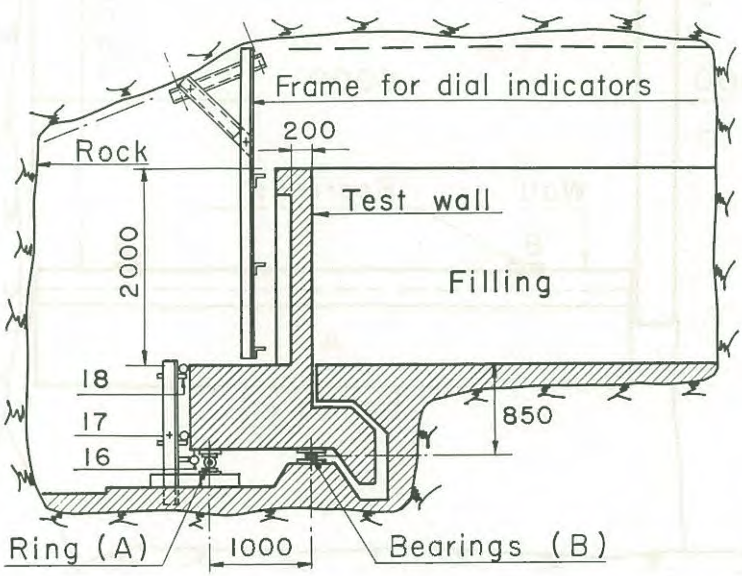


*Fig. 2 b. Section of test wall (measures in mm).*

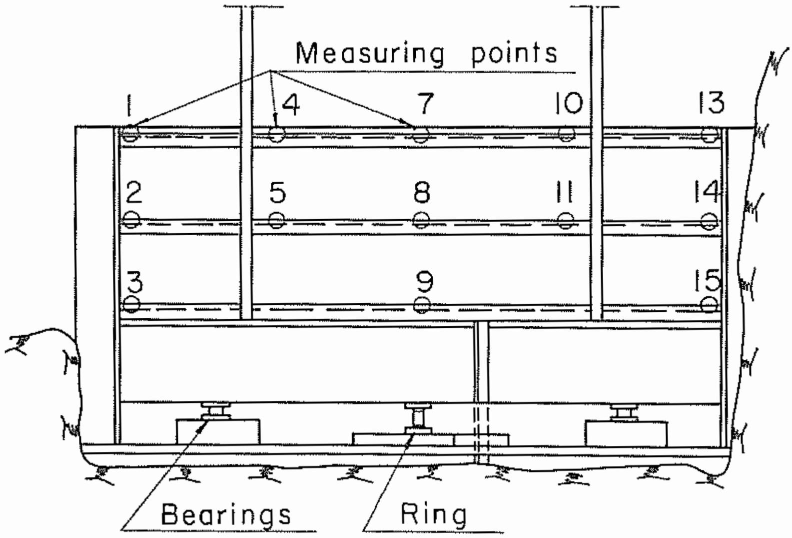


*Fig. 3. Frontal view of test wall.*

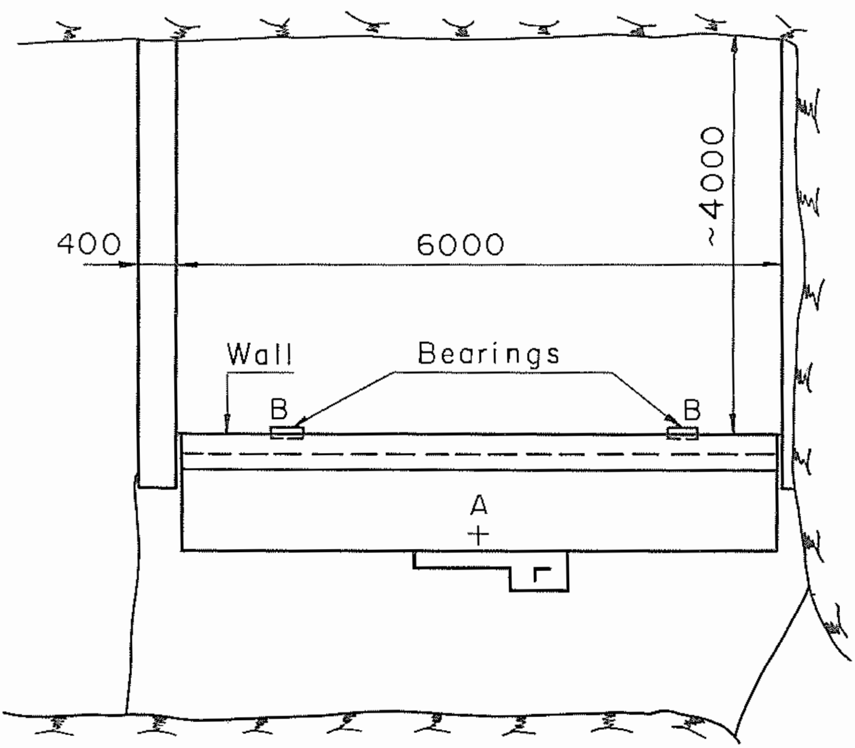


*Fig. 4. Plan of test arrangements. Figures in mm.*

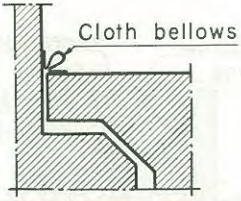


*Fig. 5. Cloth bellows between front wall and floor and between front wall and side walls.*

In the case of the experiments carried out by TERZAGHI (1930), the test arrangement had been so devised that the wall could be subject to arbitrary movements. However, with the resources available at the beginning for the actual test, it was not possible to make similar arrangements and, instead, it was decided to confine efforts to the investigation of the earth pressure against a wall displaying normal yielding movement. For this reason the thickness of the wall was fixed to one-tenth of the height (*i.e.* 0.2 m).

The wall was of reinforced concrete and in the shape of a cantilever retaining wall. The earth filling, however, was only in contact with the vertical side of the wall; this latter being cast against sides faced with wallboard so as to obtain as smooth a surface as possible. The base plate was supported at three points, *viz.*, two bearings at $B$ and a third point at $A$, where an instrument recording the reaction pressure was placed. The axis of rotation of the bearings lied in the plane formed by the wall surface facing the earth. The friction forces along the wall produced no moment around the bearings and thus had no direct effect on the support reaction measured at point $A$. An indirect effect was, however, caused by the influence on the magnitude of the earth pressure.

The retaining wall could travel freely between the side walls. The gap between these and the wall and between the floor and the front wall was covered with a strip of asphalt-impregnated cloth bellows (provided with a fold to allow free movement of the wall) to prevent small stones from falling in and becoming wedged between the surfaces (Fig. 5).

The side walls were of concrete; the one cast directly against rock and the other, of thickness 0.4 m, fixed in rock at the base. This ensured that the earth mass would not be subject to any appreciable lateral deformation.

The apparatus for measuring the reaction pressure at $A$ consisted of a sturdy steel ring (Figs. 6 and 7). The internal deflexion of the ring was measured by two so-called microcators of a type manufactured by C. E. Johansson, Eskilstuna. At a force of 13 tons the ring could be deflected 0.02 mm, which gave 100 divisions on the instrument scale. Readings were taken in half divisions, and the force could thus be measured to an accuracy of 0.065 tons.

The instrument was calibrated both in a press and by applying water pressure to the wall. In that the instrument was relatively sensitive to eccentric deformations and that it was not possible to predetermine the eccentricity arising during manufacture, the calibration obtained as a result of the water pressure

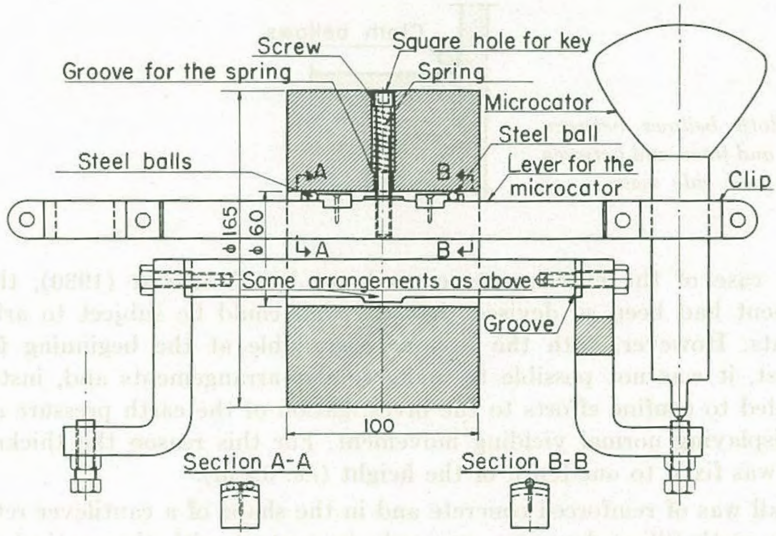


*Fig. 6. Section of pressure ring with microcator. Figures in mm.*

tests was used as the basis for evaluating the measurement values. The relation between the average readings on the microcators and the calculated load on the pressure ring is shown in Fig. 8.

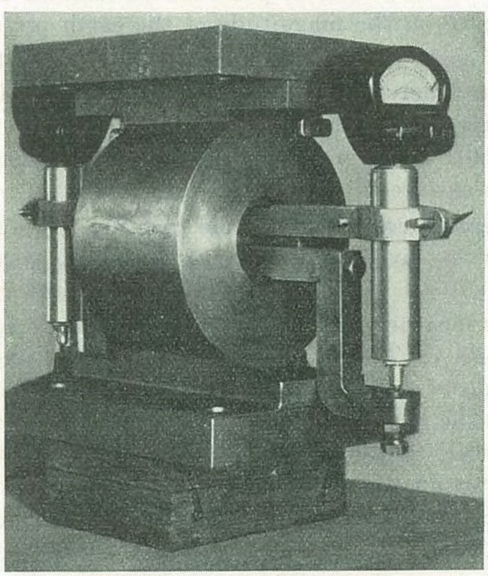

*Fig. 7. Photo of pressure ring with microcators.*

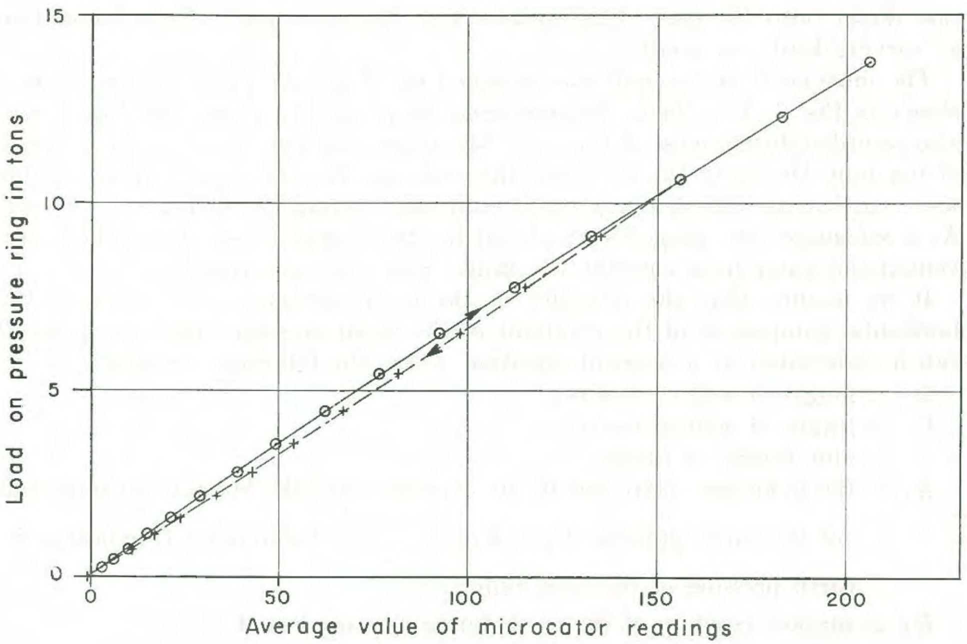


*Fig. 8. Calibration of the pressure ring by waterload on the wall.*

Between the one bearing plate of the ring and the support plate facing the concrete wall were placed two steel plates of thickness 1 mm. These could be removed when the load on the ring was relieved with the aid of jacks. In this way the wall could be made to move from the filling, at the top of the wall 5.7 mm and at the base of the wall 1.7 mm.

Temperature had a great effect on the measuring instruments. The inner diameter of the ring—that distance over which changes in length were measured —was 60 mm. A temperature change of 1° C resulted in a change in length of $1.2 \cdot 10^{-5} \cdot 60 = 0.00072$ mm, or 3.6 % of the total recording range and which means about 10 % of the reaction caused by active earth pressure. True, the extension of the ring should to some extent be compensated by a corresponding extension of the instruments but, since the ring is very thick and the instruments consist of thin material, we get a phase displacement of a size which is difficult to determine within accurate limits. With a view to eliminating these errors to the greatest possible extent, both the ring and the measuring instrument were housed in thermally insulated box. Using an electric element and a Sunvic thermostat it was possible to keep the temperature constant to within ± 0.2° C. As a check on the automatic control the temperature was also read with an ordinary mercury thermometer graduated in divisions of 0.1° C.

The entire test device was housed in a rock shelter. The bearings and the measuring ring were mounted on concrete supports which, in their turn, were cast direct onto the rock. This ensured that the movement of the foundation at varying load was small.

The movement of the wall was recorded by 15 gauges placed in the manner shown in Fig. 3. As a check, the movement at points 16, 17 and 18 (Fig. 2) was also recorded during most of the tests. Measurements were taken to an accuracy of 0.01 mm. Owing to the fact that the rock was fissured, considerable trouble was experienced with dripping water, especially during the spring and autumn. As a safeguard, the gauges were placed in transparent plastic bags which prevented the water from entering the gauges and causing corrosion.

If we assume that the intensity of the earth pressure varies linearly, the horizontal component of the resultant of the earth pressure about the point $B$ can be calculated by a moment equation. Using the following notations:

$h$ = height of wall (2 metres)
$l$ = length of wall (6 metres)
$\gamma$ = unit weight of filling
$K_E$ = the unknown coefficient in the expression for the horizontal component of the earth pressure $E_E = K_E \cdot \dfrac{\gamma \cdot h^2 \cdot l}{2}$, where index $E$ indicates the earth pressure of the back filling
$R_E$ = support reaction of the earth filling measured at $A$
$a$ = distance between $A$ and $B$ (1 metre)

we get

$$R_E \cdot a = K_E \cdot \frac{\gamma h^2}{2} \cdot l \left(\frac{h}{3} + 0.85\right)$$

from which

$$K_E = R_E \cdot \frac{2a}{l \cdot h^3 \left(0.333 + \dfrac{0.85}{h}\right) \cdot \gamma}$$

or, if all figures are measured in metric tons and metres,

$$K_E = 0.055 \cdot \frac{R_E}{\gamma}$$

In the case of tests with an overload of $q$ t/m$^2$ it is assumed—as per the classical theory for earth pressure—that the earth pressure intensity is constant and uniformly distributed over the height of the wall. Consequently, the corresponding earth pressure coefficient can be calculated as per the following formula

$$K_q = R_q \cdot \frac{a}{q \cdot l \cdot h \left(\dfrac{h}{2} + e\right)} = 0.045 \cdot \frac{R_q}{q}$$

It may seem that uncertainties arise as a result of these assumptions as to the variation in the pressure intensity or, conversely, the position of the pressure resultant. Admittedly, Terzaghi's tests show that the resultant is near the lower third point when earth pressure is exerted, but this factor has not been decisive when choosing the method of measuring. The most important factor was considered to be the establishment of the overturning moment since this is determinant for the stability of the wall against tilting and, in the case of retaining walls without counterforts, for the wall dimensions. Since, in practice, it is usual to calculate on the basis of triangularly distributed earth pressure, it is suitable to work out a triangular intensity corresponding to the derived moment. Furthermore, from the point of view of measuring technique it was of advantage to confine the measurements to one point only.

### 3 b. Influence of Friction along Front Wall

The friction between the earth mass and the wall influences not only the direction of the resultant of the earth pressure but, in addition, its magnitude. The table below shows how the $K$ value as defined above varies with the angle of friction $\delta$ between the earth mass and the wall when the friction theory is used to determine the earth pressure. The resultant of the earth pressure can then be written:

$$E_a = \lambda_a \cdot \gamma \cdot h^2/2$$

| $\delta$ | $\varphi = 30°$ | | | $\varphi = 40°$ | | |
|---|---|---|---|---|---|---|
| | $\lambda_a$ | $K = \lambda_a \cdot \cos\delta$ | $K_{\delta=0}/K_\delta$ | $\lambda_a$ | $K = \lambda_a \cdot \cos\delta$ | $K_{\delta=0}/K_\delta$ |
| 0° | 0.333 | 0.333 | 1.00 | 0.217 | 0.217 | 1.00 |
| 10° | 0.308 | 0.304 | 1.10 | 0.204 | 0.201 | 1.08 |
| 20° | 0.297 | 0.279 | 1.19 | 0.199 | 0.187 | 1.16 |
| 30° | | | | 0.201 | 0.174 | 1.25 |

The coefficient of the earth pressure at rest, 0.4-0.5, is considered to be adequately defined as long as there is no friction—or shearing stresses—between the wall and the earth mass. The values determined from Kjellman's experiments were based on the case where no shearing stresses are set up along the sides of the specimen body. Terzaghi's experiments resulted in a $K$-value of 0.405 when $\operatorname{tg}\delta = 0.4$, *i.e.*, $\delta = 21°\,20'$.

As can be seen from the table above, the friction between the wall and the earth mass reduces the horizontal component of the resultant of the earth pressure when the wall friction $\delta$ increases. If—as is likely in the tests—friction occurs, it is improbable that the theoretical earth pressure at rest will be reached. The angle of friction $\delta$ is generally considered to lie between 0 and $\frac{\varphi}{2}$. However, in the case of a low wall, $\delta$ may be smaller than with a high wall. The higher a wall, the greater is as a rule the absolute value of compression of the earth due to its own weight. First at a certain wall height there will, depending on the circumstances, be a slip and thus fully mobilized friction between the earth and the wall.

However, as has been mentioned, the object of the tests was to determine the magnitude of the horizontal component of the earth pressure acting on the wall. Even if this component—owing to the friction along the wall—should prove to be smaller than the earth pressure at rest value $K = 0.4$ à $0.5$, we are generally erring on the safe side by using the pressure value obtained as the basis for calculating the stability of the wall against overturning if the vertical component is disregarded, as is often the case.

The above line of reasoning is based on the assumption that the earth behaves as a friction soil and not as an elastic body. But it is assumed that the above gives at least some idea of the conditions involved.

### 3 c. Influence of Friction along Side Walls

The magnitude of the friction along the side walls was determined by tests. The material investigated was macadam, size 32-64 mm, and pebbles, size 16-32 mm. The angle of friction for both the macadam and the pebbles was about 40°. The surfaces of the side walls were faced with untreated metal sheeting and of metal sheeting coated with Glansoline and oil, respectively. Glansoline contains grease and graphite.

The tests apparatus consisted of a bottomless box (Figs. 9 a and 9 b) into which the stone material was poured. The box was placed on a steel plate of thickness 0.75 mm, *i.e.*, the stones were thus in direct contact with the metal. A smooth bed was provided by laying the thin steel plate on a piece of an 8 mm steel plate. The box cover was free to slide between the walls and thus rested direct on the stones. By placing various weights on the cover, it was possible to obtain varying normal pressures.

A spring balance, graduated from 0 to 150 kg, was fixed to one side of the box, and a block and 4 cables were attached to the balance. The friction was determined by increasing the tensile force until the first travel of the box could be observed. This action was repeated ten times for each normal pressure. It was found that the friction coefficient $\mu$, defined as the ratio between the horizontal tensile force ($F$) and the normal force ($N$), was practically independent of the magnitude of the normal force.

The results of the tests can be summarized as follows:

| Material | Metal surface of plate | $\mu$[1] |
|---|---|---|
| 1. Macadam | untreated ............................. | 0.54 |
| 2. ,, | lubricated with graphite and grease ....... | 0.26 |
| 3. Pebbles | untreated ............................. | 0.48 |
| 4. ,, | lubricated with graphite and grease ....... | 0.24 |

[1] Average values from ten tests.

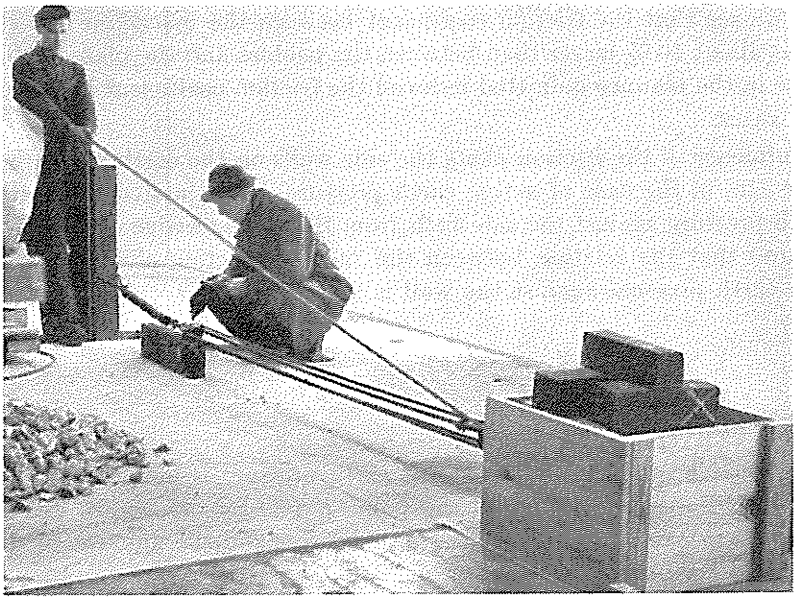

*Fig. 9 a. Photo of friction test arrangement.*

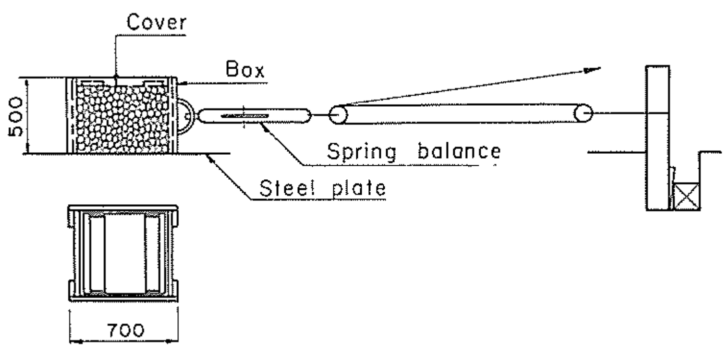


*Fig. 9 b. Friction test arrangement, in principle (measures in mm).*

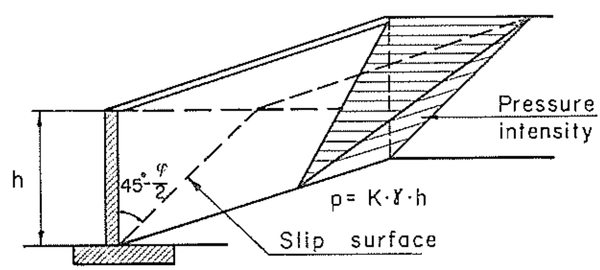


*Fig. 10. Influence on earth pressure of side wall friction.*

Thus we see that the friction coefficient between the macadam and the steel plate was about 10 % larger than between the pebbles and the plate. Lubricating the plate in the manner indicated reduced the friction coefficient by about 50 % in both cases.

The influence of the friction along the side walls on the earth pressure against the wall can, in the case of active earth pressure, be estimated by working out the earth pressure against the side walls from the sliding wedge of earth (Fig. 10). It is found that the relation between the "active" friction along the side walls and the earth pressure against the wall is

$$\frac{2F}{E} = \frac{2}{3}\mu \cdot \frac{h}{l} \cdot \mathrm{tg}\left(45^\circ - \frac{\varphi}{2}\right) = 0.014$$

*i.e.*, less than 2 %.

In the case in question, however, the earth pressure is not measured directly but, instead, the moment around the point $B$ (*cf.* Fig. 2). As an analogy, the relation between the moment of the friction forces and the moment of the earth pressure will be

$$\frac{M_F}{M_E} = \frac{2}{3}\mu \frac{\frac{h}{2} + e}{\frac{h}{3} + e} \cdot \frac{h}{l} \cdot \mathrm{tg}\left(45^\circ - \frac{\varphi}{2}\right) = 0.033$$

*i.e.*, 3.3 %.

The influence of friction along the side walls is thus so slight that it can be neglected.

## 4. Earth Pressure from Macadam

### 4 a. Properties of Macadam Used

The material investigated consisted, as mentioned above, of macadam of size 32-64 mm. The unit weight $\gamma$ was determined by weighing the macadam in a container of known volume, and was found to be 1.34 t/m$^3$ in loose state. The angle of friction of the material was determined in two ways, *viz.*, by measuring the angle of repose and by shear tests in the 50 cm compressometer of the Swedish Geotechnical Institute (Kjellman and Jakobson, 1955). The natural angle of repose was found to be 40°.

The following results were obtained from the shear tests at the Institute. The normal pressure is designated $\sigma$, in kg/cm$^2$, and the corresponding value for the shear strength $\tau$, in kg/cm$^2$. The results are not corrected for dilatancy.

| | | | | |
|---|---|---|---|---|
| | $\sigma$ | 0.91 | 1.67 | 2.45 |
| First test<br>Second test | $\tau_1$<br>$\tau_2$ | 0.81<br>0.80 | 1.36<br>1.31 | 1.99<br>2.00 |
| | $\tau_{av}$ | 0.81 | 1.34 | 2.00 |
| | $\frac{\tau_{av}}{\sigma}$ | 0.89 | 0.80 | 0.82 |
| | $\varphi = \text{arctg}\left(\frac{\tau_{av}}{\sigma}\right)$ | 41°.5 | 38°.6 | 39°.1 |

The average value was thus 39°.7, *i.e.*, about 40°.

Most of the tests showed compression, and a minority swelling. Thus the results obtained in the shear tests can, on an average, be assumed to be relatively correct values, even if correction for dilatancy is not made.

The normal pressure in the stone filling used for the tests is maximum $2.00 \times 1.34 = 2.68$ t/m², *i.e.*, 0.27 kg/cm². It will thus be seen that the normal pressures used in conjunction with the shear tests are appreciably larger than those under review and that consequently, to be applicable to the wall tests, the angle of friction of 40° assumes that the relation between $\tau$ and $\sigma$ is linear.

## 4 b. Earth Pressure against a Normally Yielding Wall from Macadam and from Overload on Macadam

*Test No. 1—Earth Pressure from Back Filling*

The back filling was in principle arranged in the manner shown in Fig. 11 a. The top surface of the filling was horizontal and flush with the top edge of the wall. The test covered a period of two months during which the load on the ring at *A*, Fig. 2, and the deflections of the wall were continually measured.

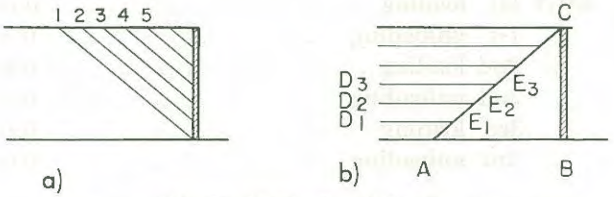


*Fig. 11. Methods of filling.*

Some of the results are shown in the table of test No. 1 (Appendix 1). As a rule, complete readings were taken at all the points of measnrement every day. The only readings taken into account were those taken when the temperature around the measuring ring was close to 21°.5 C, *i.e.*, the prevailing temperature when the tests started.

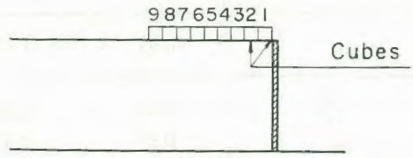


*Fig. 12. Applying the overload. Figures mark the order of applying the cube rows.*

As can be seen from the table, the first observation showed the coefficient $K$ to be 0.21. The relation between the average movement of the wall and the height of the wall was $2.1 \cdot 10^{-4}$. The reduction in the value of $K$ with time is small and may possibly be due to the slightly increased movement of the wall or to creep in the filling. However, the difference can also be attributed to temperature variations.

*Test No. 2—Earth Pressure from Overload*

The overload consisted of concrete cubes (edge length about 27 cm) filled with scrap iron, each cube weighing 63 kgs being the average of the weights of 20 cubes.

The cubes were placed on top of the macadam filling in rows parallel with the wall and starting from the back of the wall (Fig. 12). The number of cubes in each row was 23. The table of test No. 2 (Appendix 1) and Fig. 13 show how the earth pressure against the wall varied with the number of rows.

When eight rows had been added, the cubes covered an area of $2.2 \cdot 6.0 = 13.2$ m². This makes the load intensity $0.063 \cdot 8 \cdot 23/13.2 = 0.88$ t/m². Since the earth pressure increased by a relatively negligible amount after seven rows had been added, no additional load was used after the eighth row.

Loading and unloading of the cubes was performed three times. On the assumption that an overload of $q$ results in a uniformly distributed pressure on the wall of $q_h = K \cdot q$, the following values for the coefficient $K$ were obtained during the tests.

| | |
|---|---|
| After 1st loading | 0.19 |
| „ 1st unloading | 0.12 |
| „ 2nd loading | 0.22 |
| „ 2nd unloading | 0.14 |
| „ 3rd loading | 0.21 |
| „ 3rd unloading | 0.13 |

It should be noted that, after the removal of the cubes, there remained more than 60 % of the pressure on the wall caused by the overload. However, when re-applying the load, the pressure against the wall was almost exactly the same as with the first loading. The results of the tests are summarized in the table of test No. 2 (Appendix 1).

It will be seen from the table that the reading on the microcators at constant load at the beginning of the tests varied somewhat. For this reason it was diffi-

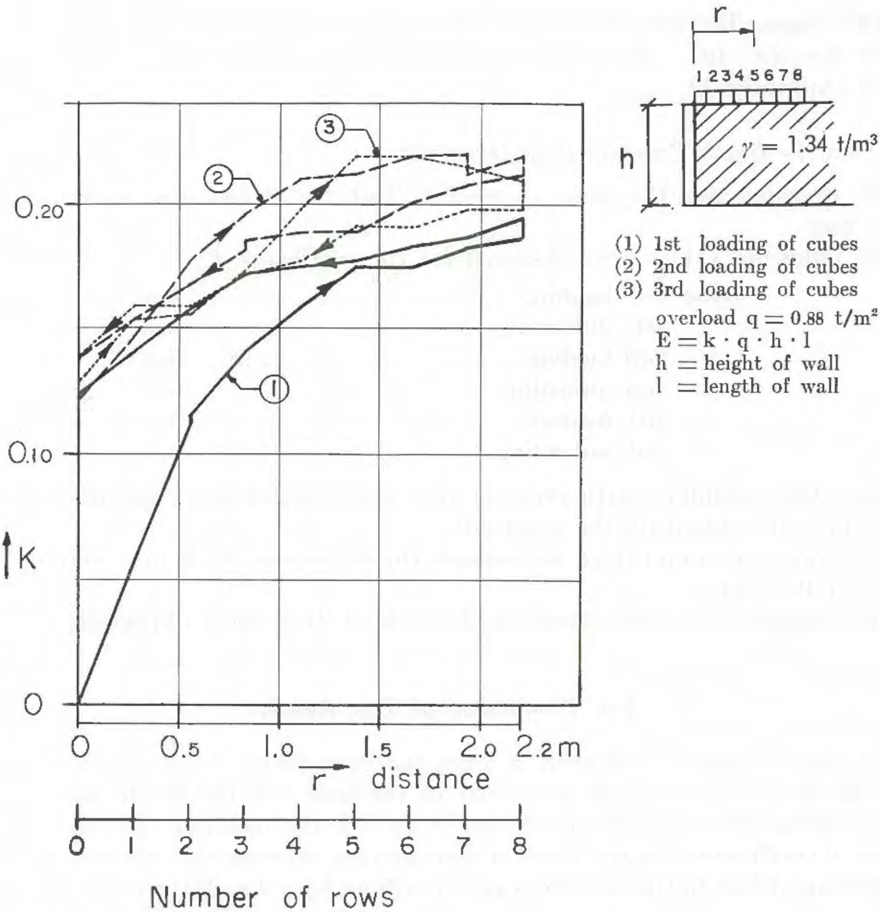


*Fig. 13. Earth pressure from overload as a function of the distribution of the overload.*

cult to establish a zero value. The zero value chosen in this case was the average of the observations made during the preceding four days. The fact that the microcator reading rose on the morning of the day of measuring may have been due to chance rise in temperature during the night.

*Test No. 3—Earth Pressure from Back Filling*

The back filling was arranged in the way shown in Fig. 11 b. First, the earth wedge *ABC* was shovelled against the wall and the slope *AC* was determined as the angle of repose. The remaining filling was then added in horizontal layers, $D_1$—$E_1$, $D_2$—$E_2$, and so on, until the full height of the filling had been reached. As had been expected, the resulting earth pressure was considerably greater than that measured during Test No. 1. The coefficient $K$ was found to be 0.30, *i.e.* a lower value than that which ought to have been obtained if the wall had been of $ABC$ shape. The relation between the average movement of the wall and the height was $2.9 \cdot 10^{-4}$. The other relevant data are shown in the table of Test No. 3 (Appendix 1).

*Test No. 4—Earth Pressure from Overload*

The overload was the same as used in Test No. 2 and was applied in the same way.

The following values were obtained for the coefficient $K$:

| | | |
|---|---|---|
| After | 1st loading | 0.22 |
| „ | 1st unloading | 0.15 |
| „ | 2nd loading | 0.24 |
| „ | 2nd unloading | 0.16 |
| „ | 3rd loading | 0.26 |
| „ | 3rd unloading | 0.17 |

Thus, the remaining earth pressure after the first unloading was 69 % of the earth pressure caused by the overload.

After the second and third unloadings, the corresponding figures were 67 and 65 %, respectively.

The other test data are shown in the table of Test No. 4 (Appendix 1).

### 4 c. Discussion of Test Results

The earth pressure coefficient $K$ from the back filling amounted to 0.21. The relation between the mean movement of the wall and the height of the wall was 1/4800. With an angle of friction of 40° for the material, the active earth pressure coefficient—on the basis of zero friction between the test wall and the filling—according to the table on page 17, will be $K_a = 0.22$. If the angle of friction between the filling and the wall is $\frac{\varphi}{2}$, *i.e.*, 20°, it will be found that $K_a = 0.19$.

In the case of active earth pressure and zero wall friction, the coefficient $K_a = \operatorname{tg}^2\left(45° - \frac{\varphi}{2}\right)$ and is rather dependent on the correct choice of value of $\varphi$. For example, if $\varphi = 45°$, $K$ becomes 0.17, *i.e.*, increasing the angle of friction by 12 % results in $K$ being reduced by 23 %. Thus an error in the angle of friction will be redoubled in the earth pressure coefficient.

When measuring the angle of friction either by measuring the angle of repose or by means of shear tests, we got a scattering in the values. In the latter case the deviations were $\Delta\varphi = +1.°8$ and $\Delta\varphi = -1.°6$ from the mean value $\varphi = 39.°9$, thus making the approximate limits

$$0.20 \leq K_a \leqq 0.22$$

It will thus be seen that the $K$-value obtained from the tests is close to the $K$-value for active earth pressure despite the movement of the wall being very small.

The experiments would thus seem to indicate that a movement of the retaining wall, amounting to an average of 1/4800 of the wall height, is sufficient to cause an earth pressure which only exceeds the active earth pressure by a negligible amount.

### 4 d. Earth Pressure from Macadam against Non-Yielding or Almost Non-Yielding Wall. Influence of Point of Time for Wall Movement

As was mentioned in the introduction to this report, it has been demonstrated experimentally that the earth pressure varies—probably continuously—between a certain value for a non-yielding wall and an active value when the wall travels some distance in a direction from the earth mass. The tests described in § 4 show only the earth pressures at a certain movement of the wall. When it was found possible with relatively simple devices to enable the wall to be subject to what were practically arbitrary movements, it was decided that the tests should be extended to include the determination of the earth pressure arising when the wall was subject to some other degrees of yield.

The first aim was to make the wall more rigid than before. This was done by giving the wall an initial movement from the filling with the aid of stays anchored in the rock. Eight stays were used; four on a level flush with the top edge of the wall and four immediately above the level of the bearings (Fig. 14). The procedure of prestressing is clear from the following reasoning.

Assume that the position of the wall under conditions of no load coincides with the line $A_1 - B_1$ in Fig. 15. According to Test No. 1 it was found that the earth pressure caused it to move to the position $A_2 - B_2$, *i.e.*, an average displacement $\Delta_0$. If, before adding the earth, we apply a prestress which moves the wall to say position $A_3 - B_3$, the travel of the wall due to earth pressure will obviously be equal to the difference between the lines $A_2 - B_2$ and $A_3 - B_3$ if the earth pressure is the same as at the movement $\Delta_0$. Now the earth pressure will, however, be greater, and consequently the final position of the wall be beyond $A_2 - B_2$, for example along the line $A_4 - B_4$. By suitable adjusting of the prestressing, the resulting movement $\Delta_1$ can be made to vary arbitrarily between the limits 0 and $\Delta_0$; it can also be made negative, *i.e.*, a movement towards the earth mass can be obtained.

However, the magnitude of the earth pressure should not be directly dependent on the movement of the wall but, instead, on the movement of the earth mass close to the wall. The latter can be calculated in the following manner.

The movement of the wall is due mainly to the vertical compression of the bearings and their horizontal travel. The elastic deformation of the wall can be neglected in this case. The movement at the top edge of the wall was $f_o = 0.60$ mm and that at the bottom edge $f_u = 0.25$ mm. Thus the average value is $f_{av} = 0.425$ mm, while the measured movement in the middle of the wall in Test No. 1 was

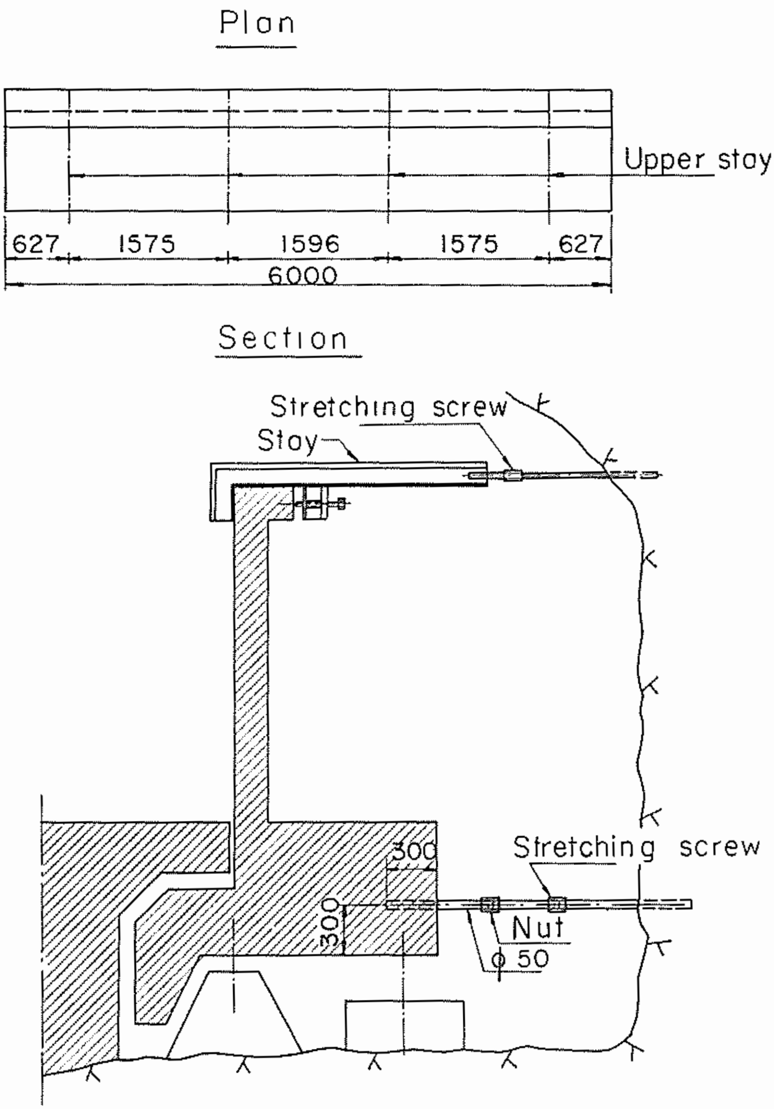


*Fig. 14. Test wall with stays for applying initial movement. Figures in mm.*

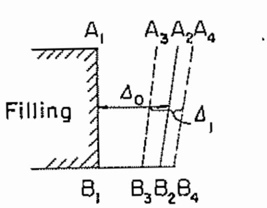


*Fig. 15. Influence of initial movement of the wall caused by prestressed stays.*

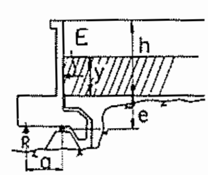


*Fig. 16. Symbols used for calculation of the movement of the earth mass.*

$f_m = 0.40$ mm. Consequently, for the purpose of the following calculations it has been assumed that the wall itself is infinitely rigid.

Using the following notations, *cf.* Fig. 16,

$y$ = height of filling

$E = K \cdot \gamma \cdot y^2 \cdot l/2$ = resultant earth pressure against the wall

$R$ = reaction pressure on the ring

$f_o$ = movement at top edge of wall

$f_u$ = movement at bottom edge of wall

$\omega$ = compression of ring, incl. reverse movement of the bearing

$\alpha$ = angular change of wall depending on $\omega$

it is found that

$$R = \frac{1}{a} \cdot E \cdot \left(\frac{y}{3} + e\right)$$

$$\omega = k_1 \cdot R$$

$$\alpha = \frac{\omega}{a} = k_1 \cdot \frac{R}{a}$$

where $k_1$ is a constant expressing vertical deformations of the bearings and the ring.

Finally we get

$$f_o = \frac{\omega}{a}(h+e) + E \cdot k_2 = E \left[ k_1 \frac{\left(\frac{y}{3} + e\right)(h+e)}{a^2} + k_2 \right]$$

$$f_u = \frac{\omega}{a} \cdot e + E \cdot k_2 = E \left[ k_1 \frac{\left(\frac{y}{3} + e\right) \cdot e}{a^2} + k_2 \right]$$

where $k_2$ is another constant expressing horizontal deformations of the bearings. The movement of the wall at height $y$ will be

$$f_y = f_u + \frac{y}{h}(f_o - f_u)$$

If we assume that the earth has been placed in horizontal layers, *i.e.*, putting $K = 0.22$ and using the basic values $y = h = 2.0$ metres, $f_o = 0.60$ mm and $f_u = 0.25$ mm, $k_1$ and $k_2$ can be solved. It will then be found that

$$k_1 = 0.033 \text{ and } k_2 = 0.025$$

The movement of the earth mass at level $z$ is

$$f_{ez} = f_{wz,\ y=h} - f_{wz,\ y=z}$$

where $f_{wz,\ y=h}$ is the movement of the wall at level $z$ and at filling height $h$ and
$f_{wz,\ y=z}$ " " " " " " " " " " " " " $z$.

We get

$$f_{ez} = 0.250 + 0.175 \cdot z - 0.043\, z^2 - 0.033\, z^3 - 0.010\, z^4$$

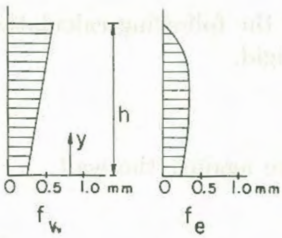


*Fig. 17. Movement of the wall ($f_w$) and corresponding deformation of the earth ($f_e$).*

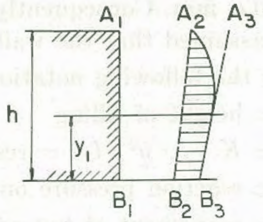


*Fig. 18. Deformation of the earth at a certain initial movement of wall by means of the stays.*

The magnitude of $f_{ez}$ is indicated in the table below, which also shows the total movement of the wall at different levels with full filling, $f_{wz,\ y=h}$, and by Fig. 17.

| $z$ m | $f_{ez}$ mm | $f_{wz}$ mm |
|---|---|---|
| 2.0 | 0.00 | 0.60 |
| 1.5 | 0.26 | 0.55 |
| 1.0 | 0.34 | 0.45 |
| 0.5 | 0.32 | 0.35 |
| 0.0 | 0.25 | 0.25 |

The area of the diagram for $f_{ez}$ in Fig. 16 is $Y_e = 0.54$ m · mm and of the diagram for $f_{wz}$ is $Y_w = 0.85$ m · mm.

Thus the average movement of an earth mass when the wall moves as the filling is applied is only 54/85, *i.e.*, 0.64 of the movement of the earth mass when the wall is kept non-yielding during the filling stage and then is allowed to move to the same total extent.

However, in the case under review, the filling was not placed in horizontal layers but, instead, the earth was allowed to reach its natural slope against the wall (see Fig. 11).

When calculating the earth movement we instead get the $f_{ez}$ values as shown below.

| $z$ m | $f_{ez}$ mm |
|---|---|
| 2.0 | 0.00 |
| 1.5 | 0.15 |
| 1.0 | 0.25 |
| 0.5 | 0.31 |
| 0.0 | 0.25 |

The area enclosed by $f_{ez}$ becomes $Y_e = 0.43$ m · mm, and conseqnently the relation $Y_e/Y_w$ becomes 0.51.

Thus, if tests are made on a wall in such a way that it can only move on an average $\Delta$ first after complete filling, this will be the same as a test in which the movement occurs simultaneously with the filling procednre and where the movements is on the average $1.5 \cdot \Delta$ à $2 \cdot \Delta$. The former value is applicable when the filling is added in horizontal layers and the latter when the filling is allowed to assume a natural slope against the wall.

The movement of the earth mass at a given prestressing of the stays is shown in Fig. 18. The line $A_1 - B_1$ indicates the original position when the wall is free from load. The line $A_2 - B_2$ shows the position reached by the wall after prestressing with the stays. The line $A_3 - B_3$ shows the position of the wall after the application of all the filling and the disconnection of the stays. The earth filling can reach the height $y_1$ before the wall is subject to any movement other than that caused by the extension of the stays on successive release of load. This extension is small and has been disregarded. When the earth exceeds the level $y_1$, the stays are completely free from load and are disconnected—automatically in the case of the upper stays which are subject only to tensile stress. The lower stays must be controlled. The wall now moves to the final position $A_3 - B_3$ and the movement of the earth is marked by the hatched area in the figure. As can be seen, this movement differs little in principle from that shown in Fig. 17. Admittedly, the difference is large at the limit stage where $y_1$ approaches $h$, but at this stage the absolute value of the earth's movement $\Delta$ is so small that no influence is noted in a diagram showing the coefficient $K$ as a function of $\Delta/h$, see, for instance, Figs. 1 and 19.

*Tests Nos. 5–8*

Tables for these tests (Appendix 1) show the results obtained when the stays of the wall were stressed to varying extents in the manner described above. The conclusive results are shown by the table reproduced in § 4 e.

*Test No. 9*

As has been mentioned previously, active earth pressure occurs when the movement of the wall is so large that failure arises in the earth mass along a slip surface. In the case of the tests in question, the maximum movement of the wall was limited to that dependent on the compression of the bearings and the measuring device and the deformations of the wall.

In spite of the fact that the agreement obtained between the measured earth pressure and earth pressure calculated on the basis of the angle of friction was relatively good—although the measured value of $K$ seemed to be a little large—we wished to obtain a value for the earth pressure when the movement of the wall was greater than in the case of Test No. 1. In this test the reaction force at $A$ was in the test measured by a pressure cell, type C. E. Johansson, Eskilstuna (the pressure ring in the macadam tests was thus not used here).

This was achieved by placing a jack under the pressure cell and allowing the jack to be compressed. The following results were obtained:

| | |
|---|---|
| Movement of wall at top edge ...... | 19.6 mm |
| „ „ „ „ bottom edge ... | 6.5 mm |
| Average movement of wall .......... | 13.1 mm |

Ratio between average movement and height of wall $= \frac{1}{154}$, *i.e.* $65 \cdot 10^{-4}$.

The following values were read on the pressure cell ($T$).

| Date (1949) | $T$ | $K$ |
|---|---|---|
| 7/2 | 0 | 0 |
| 20/2 | 3.80 | 0.16 |
| 21/2 | 4.55 | 0.19 |
| 23/2 | 4.65 | 0.19 |

The low value recorded on 20/2 was probably due to two causes, *viz.*: the filling close to the wall had settled and perhaps the friction in the bearings, *etc.*, had not been completely relieved. On 21/2 the filling had been completed and the friction had obviously been reduced. The difference between the readings on 21/2 and 23/2 is so small that the reading obtained on the latter date can be accepted as definite.

The coefficient of the horizontal component of the active earth pressure should thus be $K_a = 0.19$. The corresponding angle of friction ($\varphi$) of the material is shown in the table below, which also indicates the effect of varying degrees of wall friction ($\delta$). The table has been compiled with the aid of Krey's earth pressure tables.

| $K$ | $\delta°$ | $\varphi°$ |
|---|---|---|
| 0.19 | 0 | 42.8 |
| 0.19 | 10 | 41.1 |
| 0.19 | 20 | 39.6 |
| 0.19 | 30 | 37.9 |

The influence of the wall friction is thus not much larger than the uncertainty involved when determining the angle of friction of the material.

### 4 e. Summary of Test Results for Macadam

The results of the tests with macadam can be summarized as is shown in the tables below. The coefficient $K$ is shown as a function of the ratio between the average movement of the wall and the height of the wall. Table 1 shows the values obtained immediately after the tests were finished, and Table 2 the values obtained $D$ days (about 1 week) later. The movement, $f_{av}$, has been allowed to take place simultaneously with the filling procedure and is designated as negative when towards the earth mass and positive when away from it.

*Table 1.*

| Test No. | $K$ | $\frac{f_{av}}{h} \cdot 10^4$ |
|---|---|---|
| 6 | 0.38 | — 0.75 |
| 7 | 0.35 | — 0.15 |
| 8 | 0.35 | — 0.10 |
| 5 | 0.28 | + 1.15 |
| 1 | 0.22 | + 2.10 |
| 9 | 0.19 | + 65 |

*Table 2.*

| Test No. | $K$ | $\frac{f_{av}}{h} \cdot 10^4$ | $D$ |
|---|---|---|---|
| 6 | — | — | — |
| 7 | 0.34 | — 0.40 | 6 |
| 8 | 0.33 | — 0.25 | 7 |
| 5 | — | — | — |
| 1 | 0.22 | + 2.3 | 8 |
| 9 | — | — | — |

The relation between $K$ and $f_{av}$ is also shown in Fig. 19.

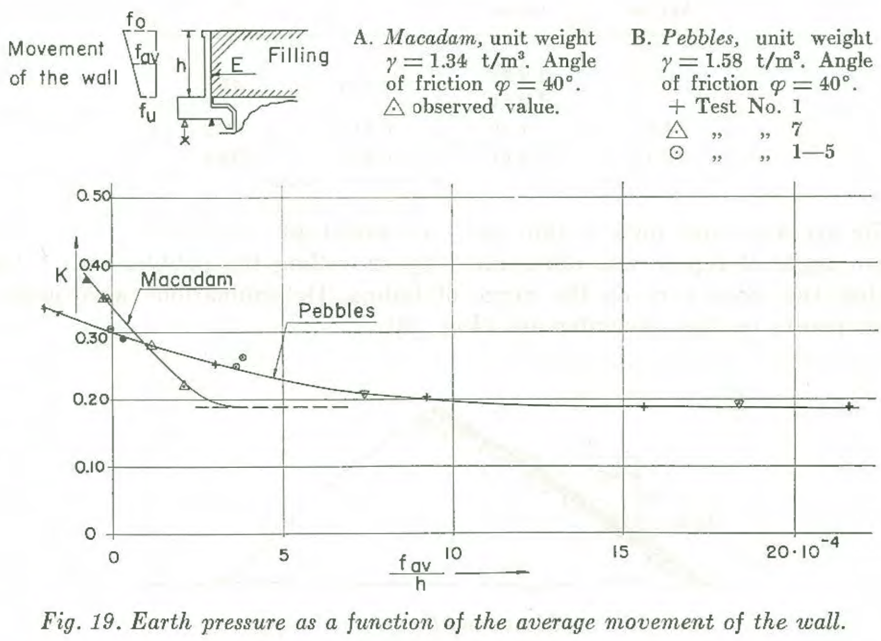


*Fig. 19. Earth pressure as a function of the average movement of the wall.*

# 5. Earth Pressure from Pebbles

## 5 a. Properties of Pebbles Used

Since the difference between earth pressure at rest and active earth pressure could be expected to be especially pronounced in the case of macadam, the original intention was that the investigation should be confined to this material. Later, when it had been found, among other things, that the measured earth pressure at rest was considerably lower than had been expected, it was considered desirable to augment the investigation by tests on another common material which could be expected to have other values of the material properties, such as concerns modulus of elasticity and internal friction. This means that sand or gravel would have been the most interesting materials to investigate but, since the shelter was very moist—with water running from the walls and ceiling at times—these soils were out of the question. As is known, these materials are supposed to be more or less cohesive when damp and thus the results could be expected to be misleading. Instead, we chose pebbles of size 16-32 mm. The stones were rounded and, in general, oval. The unit weight was determined as 1.58 $t/m^3$, in loose state.

As had been the case with the macadam, the angle of friction was determined by shear tests carried out at the Swedish Geotechnical Institute and by measuring the angle of repose.

The table below shows the results of the shear tests.

| $\sigma$ kg/cm² | $\tau$ kg/cm² | $\tau/\sigma$ | $\varphi°$ |
|---|---|---|---|
| 0.91 | {0.81<br>0.82 | 0.896 | 41.9 |
| 1.67 | 1.36 | 0.814 | 39.1 |
| 2.45 | 2.01 | 0.820 | 39.4 |

The average value for $\varphi$ is thus 40.°1, *i.e.* about 40°.

The angle of repose was determined by shovelling the pebbles into a heap, so that the sides were on the verge of failing. Determinations were made at seven points on the circumference (Fig. 20).

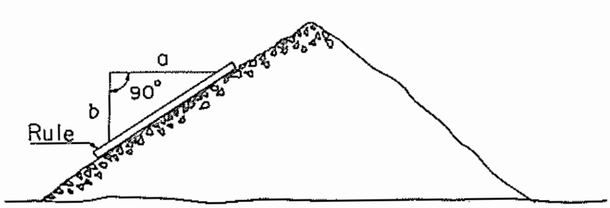


*Fig. 20. Measuring the angle of repose.*

| Test No. | $a$ mm | $b$ mm | $b/a$ | $\varphi°$ |
|---|---|---|---|---|
| 1 | 660 | 540 | 0.818 | 39.3 |
| 2 | 690 | 532 | 0.772 | 37.7 |
| 3 | 690 | 542 | 0.785 | 38.2 |
| 4 | 618 | 555 | 0.898 | 42.0 |
| 5 | 680 | 590 | 0.868 | 41.0 |
| 6 | 648 | 590 | 0.910 | 42.3 |
| 7 | 696 | 570 | 0.820 | 39.3 |
| Average | | | 0.838 | 40.0 |

Most of the shear tests showed swelling of the material. Thus the results corrected for dilatancy ought to have been lower than the figures above. However, as the angle of repose was as high as 40°, no reduction of the angle has been made in the following treatment. (*Cf.* BISHOP, 1950.)

The results of the experiments carried out with pebbles are shown in table form in Appendix 2 and in § 5 c.

## 5 b. Tests

Test data can be found in Appendix 2.

In *Tests Nos. 1 and 5* the wall was allowed to move as the filling was applied.

In *Tests Nos. 2, 3 and 4* the stays of the wall had been prestressed to varying degrees and the earth pressure was therefore obtained as a function of different movements of the wall which were less than those obtained in the case of Tests Nos. 1 and 5.

In *Tests Nos. 6 and 7* a jack was placed under the cell, so that the wall could be made to move both towards and away from the filling. This gave a number of relations between the movement and the earth pressure. Since the filling behind the wall was not removed after Test No. 5, the cell was unloaded by means of a jack placed beside the cell. The whole wall was lifted under the base so that another, small jack could be placed under the cell. As a result, the earth pressure against the wall rose slightly towards the passive value. The pressure measured at $A$ is the total load, and the force determining the magnitude of the earth pressure was calculated by reducing the reading shown on the cell by 7.2 tons, which is the calculated reaction of the weight of the wall.

A series of values obtained on movement from the filling are given in the tables for Tests 6 and 7 (Appendix 2).

Since the displacement $\Delta$ was not determined in relation to any basic value, the curve showing the relation between $K$ and $\Delta/h$ has been so arranged that the $K$-value for earth pressure at rest obtained from earlier tests—Nos. 3.

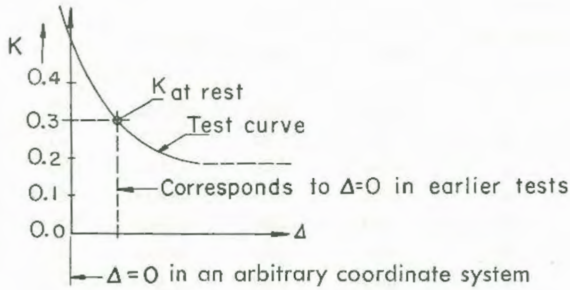


*Fig. 21. Fitting curve* $K = f\left(\frac{\Delta}{h}\right)$ *from Tests Nos. 6 and 7 to earlier tests in the report.*

and 4—*i.e.*, $K = 0.30$, coincides with the same value obtained during the test. The procedure is shown in Fig. 21.

As was mentioned in § 4 e, the displacements described in Test No. 6 cannot be directly compared with those obtained from Tests Nos. 1-5. When making a comparison, the average movement in Tests Nos. 6 and 7 was multiplied by 2.0, and this calculated value is accounted for separately in the summary below.

### 5 c. Summary of Test Results for Pebbles

The results of the tests with pebbles can be summarized in the manner used in the table below. The coefficient $K$ for the horizontal component of the pressure resultant is expressed as a function of the relation between the mean movement of the wall and the height of the wall, when the movement occurs simultaneously with the filling procedure. The movements measured in Tests Nos. 6 and 7 therefore have been multiplied by 2, *cf.* p. 29, to enable a comparison with the results of Tests Nos. 1–5.

On comparing point $K = 0.25$, $f_{av}/h = 1.5 \cdot 10^{-4}$ from Test No. 6 with point $K = 0.25$, $f_{av}/h = + 3.6 \cdot 10^{-4}$, it is found that the factor by which the values for $f_{av}/h$ in Tests Nos. 6 and 7 must be multiplied to enable a fair comparison with the other tests is $3.6/1.5 = 2.4$ instead of 2.0 indicated theoretically above. However, demands as to accuracy must be kept within reasonable limits. It is of course impossible to let $K$ be a function of the mean movement of the wall alone. The magnitude of the movement at different levels should also have a certain effect. In the tests performed the movement at the base of the wall has generally been smaller than that at the top, and the relation between them has approximately corresponded to that which can be expected in practice. However, judging by the satisfactory manuer in which the test values can be arranged in a curve showing $K$ as a function of $f_{av}/h$, it would seem that the influence of the turn of the wall is relatively small.

| Test No. | $K$ | $f_{av}/h \cdot 10^{-4}$ | |
|---|---|---|---|
| | | Movement of wall simultaneously with filling procedure | Movement of wall after filling has been applied |
| 6 | 0.34 | (— 2.0) | — 1.0 |
| 3 | 0.31 | — 0.05 | |
| 4 | 0.31 | — 0.05 | |
| 2 | 0.29 | 0.30 | |
| 6 | 0.25 | (3.0) | 1.5 |
| 1 | 0.26 | 3.8 | |
| 5 | 0.25 | 3.6 | |
| 6 | 0.20 | (9.2) | 4.6 |
| 6 | 0.19 | (15.6) | 7.8 |
| 6 | 0.19 | (21.6) | 10.8 |
| 6 | 0.18 | (27.2) | 13.6 |

The figures in parentheses are the converted values from Test No. 6.

In the diagram in Fig. 19 the coefficient $K$ has been expressed as a function of the mean movement of the wall both for macadam and pebbles. As can be seen, the materials have fairly different properties. Pebbles thus require a considerably larger movement of the wall than macadam to bring about active earth pressure.

## Appendix 1

### *Earth Pressure from Macadam*

Test No. 1

$\gamma = 1.34$ t/m³
$E = K_E \cdot \gamma \cdot h^2/2$

| Number of days | Temperature: in rock shelter $t_y$ °C | Temperature: around pressure cell $t_i$ °C | Wall movement $f_o$ (0.01 mm) | Wall movement $f_m$ (0.01 mm) | Wall movement $f_u$ (0.01 mm) | $K_E$ | $\frac{f_{av}}{h} \cdot 10^4$ |
|---|---|---|---|---|---|---|---|
| 0 | 9.9 | | 0 | 0 | 0 | 0 | 0 |
| 1 | 11.7 | 21.0 | 60 | 40 | 25 | 0.222 | 2.1 |
| 9 | 8.6 | 21.5 | 70 | 40 | 23 | 0.218 | 2.2 |
| 19 | 9.9 | 21.5 | 73 | 42 | 23 | 0.212 | 2.3 |
| 31 | 11.6 | 21.5 | 75 | 44 | 23 | 0.209 | 2.4 |
| 42 | 10.7 | 21.6 | 79 | 44 | 21 | 0.201 | 2.4 |
| 52 | 10.8 | 21.7 | 77 | 46 | 21 | 0.205 | 2.4 |
| 59 | 12.2 | 21.5 | 75 | 46 | 20 | 0.209 | 2.4 |

### *Earth Pressure from Overload ($q = 0.88$ t/m²) on macadam*

Test No. 2

$E = K_q \cdot q \cdot h$

| Number of days | Temperature: in rock shelter $t_y$ °C | Temperature: around pressure cell $t_i$ °C | $f_o$ (0.01 mm) | $f_m$ (0.01 mm) | $f_u$ (0.01 mm) | $K_q$ | $\frac{f_{av}}{h} \cdot 10^4$ | Number of cube rows |
|---|---|---|---|---|---|---|---|---|
| —4 | 12.7 | 21.8 | | | | | | 0 |
| —3 | 12.2 | 21.5 | | | | | | 0 |
| —1 | 11.7 | 20.8 | | | | | | 0 |
| 0 | 14.2 | 21.0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \| | 14.7 | 21.6 | 10 | 7 | 2 | 0.057 | 0.3 | 1 |
| \| | 15.0 | 20.4 | 22 | 15 | 6 | 0.112 | 0.7 | 2 |
| \| | 14.9 | 20.6 | 24 | 17 | 6 | 0.115 | 0.8 | 2 |
| \| | 14.9 | 20.2 | 31 | 21 | 9 | 0.141 | 1.0 | 3 |
| \| | 15.0 | 20.8 | 35 | 24 | 10 | 0.159 | 1.2 | 4 |
| \| | 15.3 | — | 38 | 27 | 12 | 0.175 | 1.3 | 5 |
| \| | 15.3 | 21.3 | 40 | 28 | 13 | 0.175 | 1.4 | 6 |
| \| | 15.5 | 20.8 | 40 | 28 | 13 | 0.183 | 1.4 | 7 |
| 0 | 15.5 | 20.5 | 40 | 28 | 13 | 0.187 | 1.4 | 8 |
| 1 | 12.5 | 20.8 | 44 | 27 | 12 | 0.191 | 1.4 | 8 |
| 2 | 12.0 | 21.8 | 44 | 28 | 12 | 0.205 | 1.4 | 8 |
| 3 | 11.9 | 21.5 | 45 | 29 | 13 | 0.205 | 1.5 | 8 |
| 6 | 12.4 | 20.3 | 46 | 29 | 13 | 0.196 | 1.5 | 8 |

Test No. 2 (continued)

| Number of days | Temperature in rock shelter $t_y$ °C | Temperature around pressure cell $t_i$ °C | $f_o$ (0.01 mm) | $f_m$ (0.01 mm) | $f_u$ (0.01 mm) | $K_q$ | $\frac{f_{av}}{h} \cdot 10^4$ | Number of cube rows |
|---|---|---|---|---|---|---|---|---|
| 7 | 13.0 | 19.4 | 46 | 29 | 13 | 0.183 | 1.5 | 8 |
| 8 | — | 20.2 | — | — | — | 0.196 | — | 8 |
| | 13.6 | 20.1 | 46 | 30 | 14 | 0.195 | 1.5 | 8 |
| | 15.5 | 20.2 | 44 | 31 | 15 | 0.187 | 1.5 | 7 |
| | 15.8 | 20.2 | 43 | 32 | 17 | 0.183 | 1.6 | 6 |
| | 16.3 | 20.6 | 42 | 32 | 18 | 0.177 | 1.6 | 5 |
| | 16.8 | 20.2 | 41 | 31 | 17 | 0.175 | 1.5 | 4 |
| | 16.8 | 20.2 | 40 | 31 | 18 | 0.172 | 1.5 | 3 |
| | 16.8 | 20.2 | 39 | 31 | 18 | 0.158 | 1.5 | 2 |
| | 16.8 | 20.7 | 38 | 29 | 17 | 0.141 | 1.4 | 1 |
| 8 | 16.0 | 20.2 | 36 | 28 | 17 | 0.124 | 1.4 | 0 |
| 9 | 12.8 | 20.2 | 35 | 24 | 15 | 0.117 | 1.3 | 0 |
| 10 | 12.3 | 20.5 | 34 | 23 | 15 | 0.117 | 1.2 | 0 |
| 11 | 12.3 | 20.5 | 34 | 24 | 15 | 0.118 | 1.2 | 0 |
| 13 | 12.3 | 20.6 | 34 | 24 | 16 | 0.121 | 1.3 | 0 |
| 14 | 12.6 | 20.1 | 34 | 24 | 16 | 0.121 | 1.3 | 0 |
| 15 | 13.8 | 20.8 | 33 | 26 | 17 | 0.115 | 1.3 | 0 |
| 16 | 13.5 | 20.5 | 33 | 25 | 16 | 0.122 | 1.3 | 0 |
| | 16.3 | 20.5 | 35 | 29 | 18 | 0.147 | 1.4 | 1 |
| | 16.7 | 20.5 | 39 | 32 | 19 | 0.177 | 1.5 | 2 |
| | 16.5 | 20.3 | 42 | 35 | 20 | 0.196 | 1.6 | 3 |
| | 17.5 | 20.5 | 44 | 37 | 21 | 0.210 | 1.7 | 4 |
| | 17.8 | 20.5 | 44 | 38 | 21 | 0.212 | 1.7 | 5 |
| | 18.0 | 20.4 | 44 | 38 | 22 | 0.219 | 1.8 | 6 |
| | 18.2 | 20.4 | 45 | 38 | 22 | 0.220 | 1.8 | 7 |
| | 14.0 | 20.5 | — | — | — | 0.212 | — | 7 |
| 16 | 18.2 | 20.5 | 45 | 37 | 21 | 0.219 | 1.7 | 8 |
| 17 | 13.8 | 20.0 | 44 | 35 | 19 | 0.210 | 1.7 | 8 |
| 18 | 13.2 | 20.0 | 44 | 35 | 20 | 0.211 | 1.7 | 8 |
| 20 | 12.8 | 20.4 | 44 | 35 | 20 | 0.210 | 1.7 | 8 |
| 21 | 12.5 | 20.0 | 43 | 35 | 20 | 0.212 | 1.7 | 8 |
| | 14.1 | 20.0 | 42 | 36 | 21 | 0.208 | 1.7 | 7 |
| | 14.3 | 20.1 | 41 | 35 | 21 | 0.201 | 1.6 | 6 |
| | 14.5 | 21.5 | 40 | 34 | 21 | 0.189 | 1.6 | 5 |
| | 14.3 | 20.6 | 39 | 34 | 20 | 0.189 | 1.6 | 4 |
| | 14.4 | 19.9 | 38 | 33 | 20 | 0.185 | 1.5 | 3 |
| | 14.4 | 20.7 | 38 | 33 | 20 | 0.182 | 1.5 | 3 |
| | 14.5 | 20.5 | 37 | 32 | 20 | 0.168 | 1.5 | 2 |
| | 14.3 | 20.8 | 35 | 31 | 20 | 0.154 | 1.5 | 1 |
| 21 | 14.3 | 20.4 | 34 | 31 | 20 | 0.138 | 1.4 | 0 |
| 22 | 12.4 | 20.0 | 33 | 29 | 19 | 0.128 | 1.4 | 0 |

Test No. 2 (continued)

| Number of days | Temperature in rock shelter $t_y$ °C | Temperature around pressure cell $t_i$ °C | $f_o$ (0.01 mm) | $f_m$ (0.01 mm) | $f_u$ (0.01 mm) | $K_q$ | $\frac{f_{av}}{h} \cdot 10^4$ | Number of cube rows |
|---|---|---|---|---|---|---|---|---|
| 25 | 12.8 | 20.2 | 33 | 29 | 19 | 0.128 | 1.4 | 0 |
| 27 | 13.6 | 20.3 | 33 | 29 | 20 | 0.129 | 1.4 | 0 |
| | 14.5 | 20.3 | 36 | 33 | 21 | 0.154 | 1.5 | 1 |
| | 14.7 | 23.8 | 37 | 33 | 20 | 0.156 | 1.5 | 2 |
| | 14.7 | 23.4 | 37 | 34 | 20 | 0.173 | 1.5 | 3 |
| | 14.8 | 22.0 | 38 | 34 | 21 | 0.198 | 1.6 | 4 |
| | 15.0 | 20.3 | 38 | 35 | 21 | 0.219 | 1.6 | 5 |
| | 14.8 | 20.0 | 38 | 35 | 21 | 0.219 | 1.6 | 6 |
| | 14.6 | 20.2 | 38 | 35 | 21 | 0.215 | 1.6 | 7 |
| | 15.0 | 21.0 | 38 | 35 | 21 | 0.210 | 1.6 | 8 |
| 27 | — | 21.0 | — | — | — | 0.205 | — | 8 |
| 28 | 12.5 | 20.8 | 38 | 34 | 20 | 0.205 | 1.6 | 8 |
| 29 | 12.3 | 20.5 | 39 | 34 | 20 | 0.200 | 1.6 | 8 |
| 30 | 12.3 | 19.9 | 39 | 34 | 21 | 0.195 | 1.6 | 8 |
| 30 | — | — | — | — | — | 0.195 | | 8 |
| 31 | 12.3 | 19.9 | 44 | 34 | 21 | 0.195 | 1.7 | 8 |
| 32 | 12.2 | 20.3 | 45 | 34 | 21 | 0.198 | 1.7 | 8 |
| 34 | 12.2 | 20.3 | 46 | 35 | 21 | 0.198 | 1.7 | 8 |
| 35 | 12.0 | 20.3 | 46 | 35 | 21 | 0.196 | 1.7 | 8 |
| 36 | 12.0 | 20.4 | 47 | 35 | 21 | 0.198 | 1.7 | 8 |
| | 12.7 | 20.6 | 47 | 35 | 20 | 0.198 | 1.7 | 8 |
| | 14.1 | 20.1 | 46 | 36 | 21 | 0.198 | 1.7 | 7 |
| | 14.1 | 20.6 | 46 | 36 | 22 | 0.191 | 1.8 | 6 |
| | 14.1 | 20.0 | 46 | 36 | 22 | 0.191 | 1.8 | 5 |
| | 14.2 | 20.3 | 46 | 36 | 22 | 0.177 | 1.8 | 4 |
| | 14.3 | 20.4 | 45 | 35 | 22 | 0.172 | 1.7 | 3 |
| | 14.0 | 21.3 | 44 | 35 | 22 | 0.159 | 1.7 | 2 |
| | 14.2 | 20.1 | 43 | 35 | 21 | 0.159 | 1.7 | 1 |
| 36 | 14.0 | 20.1 | 41 | 34 | 22 | 0.139 | 1.6 | 0 |
| 37 | 12.0 | 20.0 | 42 | 32 | 19 | 0.131 | 1.6 | 0 |
| 38 | 12.3 | 20.4 | 42 | 31 | 19 | 0.131 | 1.6 | 0 |

*Earth pressure from macadam* $\gamma = 1.34$ t/m³

Test No. 3 $E = K_E \cdot \gamma \cdot h^2/2$

| Number of days | Temperature | | $f_o$ | $f_m$ | $f_u$ | $K_E$ | $\frac{f_{av}}{h} \cdot 10^4$ | Notes |
|---|---|---|---|---|---|---|---|---|
| | in rock shelter $t_y$ °C | around pressure cell $t_i$ °C | 0.01 mm | | | | | |
| 0 | 11.4 | 20.4 | 0 | 0 | 0 | 0 | 0 | No load |
| 4 | 11.2 | 20.2 | 96 | 51 | 26 | 0.30 | 2.9 | Full height |

*Earth pressure from overload ($q = 0.88$ t/m²) on macadam*

Test No. 4 $E = K_q \cdot q \cdot h$

| Number of days | Temperature | | $f_o$ | $f_m$ | $f_u$ | $K_q$ | $\frac{f_{av}}{h} \cdot 10^4$ | Number of cube rows |
|---|---|---|---|---|---|---|---|---|
| | in rock shelter $t_y$ °C | around pressure cell $t_i$ °C | 0.01 mm | | | | | |
| 0 | 12.2 | 20.4 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 12.2 | 20.3 | 13 | 9 | 4 | 0.06 | 0.5 | 1 |
| | 12.5 | 20.4 | 23 | 16 | 7 | 0.10 | 0.8 | 2 |
| | — | 20.5 | 30 | 21 | 10 | 0.14 | 1.0 | 3 |
| | 12.8 | 20.3 | 38 | 26 | 13 | 0.17 | 1.3 | 4 |
| | 12.8 | 20.4 | 41 | 29 | 15 | 0.19 | 1.4 | 5 |
| | 13.0 | 20.5 | 44 | 31 | 17 | 0.21 | 1.6 | 6 |
| | 13.0 | 20.4 | 45 | 32 | 17 | 0.22 | 1.6 | 7 |
| 0 | 13.1 | 20.3 | 47 | 33 | 18 | 0.22 | 1.7 | 8 |
| 3 | 11.0 | 20.5 | 54 | 36 | 18 | 0.23 | 1.8 | 8 |
| | 11.0 | 20.4 | 54 | 36 | 18 | 0.23 | 1.8 | 8 |
| | 11.0 | 20.4 | 54 | 36 | 18 | 0.22 | 1.8 | 7 |
| | 11.0 | 20.4 | 54 | 36 | 18 | 0.22 | 1.8 | 6 |
| | 11.0 | 20.4 | 53 | 35 | 18 | 0.22 | 1.8 | 5 |
| | 11.0 | 20.4 | 53 | 35 | 18 | 0.20 | 1.8 | 4 |
| | 10.7 | 20.4 | 52 | 34 | 18 | 0.19 | 1.8 | 3 |
| | 10.7 | 20.4 | 51 | 33 | 17 | 0.18 | 1.7 | 2 |
| | 10.8 | 20.4 | 51 | 33 | 17 | 0.17 | 1.7 | 1 |
| 3 | 10.7 | 20.4 | 49 | 32 | 16 | 0.15 | 1.6 | 0 |
| 4 | 10.2 | 20.4 | 51 | 32 | 15 | 0.14 | 1.7 | 0 |
| 5 | 10.0 | 20.4 | 58 | 33 | 15 | 0.12 | 1.8 | 0 |
| | 9.3 | 20.4 | 63 | 34 | 14 | 0.12 | 1.9 | 0 |
| | 9.2 | 20.5 | 73 | 38 | 15 | 0.16 | 2.1 | 1 |
| | 9.1 | 20.5 | 80 | 40 | 15 | 0.18 | 2.3 | 2 |
| | 9.1 | 20.5 | 84 | 43 | 16 | 0.20 | 2.4 | 3 |
| | 9.2 | 20.4 | 86 | 44 | 17 | 0.22 | 2.5 | 4 |
| 5 | 9.6 | 20.4 | 88 | 46 | 19 | 0.23 | 2.6 | 5 |

Test No. 4 (continued)

| Number of days | Temperature in rock shelter $t_y$ °C | Temperature around pressure cell $t_i$ °C | $f_o$ (0.01 mm) | $f_m$ (0.01 mm) | $f_u$ (0.01 mm) | $K_q$ | $\frac{f_{av}}{h} \cdot 10^4$ | Number of cube rows |
|---|---|---|---|---|---|---|---|---|
| 5 | 9.7 | 20.4 | 90 | 47 | 19 | 0.23 | 2.6 | 6 |
| 1 | 10.0 | 20.4 | 90 | 47 | 19 | 0.24 | 2.6 | 7 |
| 5 | 10.0 | 20.4 | 91 | 48 | 20 | 0.24 | 2.7 | 8 |
| 7 | 10.8 | 20.6 | 95 | 51 | 22 | 0.24 | 2.8 | 8 |
| 8 | 10.8 | 20.5 | 94 | 51 | 23 | 0.25 | 2.8 | 8 |
| 9 | 9.8 | 20.7 | 99 | 52 | 22 | 0.23 | 2.9 | 8 |
| 10 | 9.7 | 21.0 | 106 | 53 | 21 | 0.22 | 3.0 | 8 |
| | 8.8 | 20.8 | 108 | 54 | 21 | 0.22 | 3.1 | 7 |
| | 9.1 | 20.7 | 108 | 54 | 21 | 0.22 | 3.1 | 6 |
| | 9.2 | 20.7 | 108 | 54 | 20 | 0.22 | 3.1 | 5 |
| | 9.0 | 20.6 | 109 | 53 | 20 | 0.20 | 3.1 | 4 |
| | 9.3 | 20.7 | 109 | 53 | 20 | 0.20 | 3.1 | 3 |
| | 9.2 | 20.7 | 108 | 53 | 20 | 0.19 | 3.0 | 2 |
| | 9.2 | 20.7 | 108 | 52 | 19 | 0.18 | 3.0 | 1 |
| 10 | 9.1 | 20.9 | 107 | 52 | 19 | 0.16 | 3.0 | 0 |
| 11 | 9.7 | 20.6 | 105 | 51 | 20 | 0.16 | 3.0 | 0 |
| 12 | — | 20.7 | 107 | 52 | 19 | 0.15 | 3.0 | 0 |
| 14 | 9.5 | 20.8 | 108 | 52 | 19 | 0.14 | 3.0 | 0 |
| 15 | 9.1 | 20.3 | 111 | 52 | 19 | 0.13 | 3.1 | 0 |
| | 9.8 | 20.3 | 111 | 52 | 19 | 0.13 | 3.1 | 0 |
| | 9.8 | 20.3 | 119 | 56 | 20 | 0.17 | 3.3 | 1 |
| | 9.6 | 20.5 | 125 | 59 | 22 | 0.19 | 3.5 | 2 |
| | 9.6 | 20.3 | 127 | 60 | 22 | 0.20 | 3.5 | 3 |
| | 9.6 | 20.4 | 128 | 62 | 22 | 0.21 | 3.6 | 4 |
| | 9.6 | 20.4 | 130 | 62 | 23 | 0.22 | 3.6 | 5 |
| | 9.5 | 20.5 | 130 | 63 | 23 | 0.23 | 3.6 | 6 |
| | 9.6 | 20.3 | 131 | 64 | 24 | 0.24 | 3.7 | 7 |
| 15 | 9.6 | 20.4 | 136 | 67 | 25 | 0.26 | 3.8 | 8 |
| 16 | 9.8 | 20.4 | 140 | 68 | 25 | 0.25 | 3.9 | 8 |
| 17 | 9.2 | 20.3 | 147 | 69 | 25 | 0.24 | 4.0 | 8 |
| | 9.2 | 20.3 | 148 | 72 | 26 | 0.24 | 4.1 | 9 |
| | 9.3 | 20.3 | 148 | 71 | 25 | 0.24 | 4.1 | 8 |
| | 9.2 | 20.3 | 148 | 71 | 25 | 0.24 | 4.1 | 7 |
| | 9.2 | 20.3 | 147 | 71 | 25 | 0.23 | 4.1 | 6 |
| | 9.2 | 20.4 | 147 | 70 | 25 | 0.23 | 4.1 | 5 |
| | 9.0 | 20.3 | 147 | 70 | 24 | 0.22 | 4.0 | 4 |
| | 9.1 | 20.4 | 146 | 69 | 24 | 0.22 | 4.0 | 3 |
| | 8.9 | 20.5 | 146 | 68 | 23 | 0.20 | 4.0 | 2 |
| | 8.8 | 20.4 | 144 | 68 | 23 | 0.19 | 3.9 | 1 |
| 17 | 8.6 | 20.4 | 143 | 67 | 22 | 0.17 | 3.9 | 0 |
| 18 | 8.4 | 20.4 | 143 | 66 | 22 | 0.17 | 3.9 | 0 |

*Earth pressure from macadam*

Wall initially loaded by prestressed stays.

Tests Nos. 5, 6, 7 and 8

$E = K_E \cdot \gamma \cdot h^2/2$

| Test No. | Temperature in rock shelter $t_y$ °C | Temperature around pressure cell $t_i$ °C | $K_E$ | $\frac{f_{av}}{h} \cdot 10^4$ | Notes |
|---|---|---|---|---|---|
| 5 | 7.0 | 14.6 | 0 | 0 | No back filling. |
| | 8.4 | 14.6 | | | Do. Prestressed stays. |
| | 9.0 | 14.7 | | | Backfilling to full height. Prestressed stays. |
| | 9.3 | 14.8 | | | Backfilling to full height. Stays disconnected. |
| | | | 0.28 | 1.15 | Wall movement. |
| 6 | 9.2 | 14.8 | 0 | 0 | No backfilling. |
| | 9.8 | 14.2 | | | No backfilling. Prestressed stays. |
| | 10.2 | 15.5 | | | Backfilling to full height. Stays disconnected. |
| | | | 0.38 | — 0.75 | Wall movement. |
| 7 | 9.9 | 14.8 | 0 | 0 | No backfilling. |
| | 9.6 | 15.8 | | | Do. Prestressed stays. |
| | 10.6 | 14.2 | | | Backfilling to full height. Stays disconnected. |
| | 10.5 | 15.8 | | | Do. Readings after 6 days. |
| | 11.2 | 15.0 | | | Do. Readings after 12 days. |
| | | | 0.345 | — 0.15 | Wall movement, immediately. |
| | | | 0.335 | — 0.40 | Wall movement after 6 days. |
| | | | 0.335 | — 0.35 | Wall movement after 12 days. |
| 8 | 12.3 | 14.3 | 0 | 0 | No backfilling. |
| | 11.8 | 16.2 | | | No backfilling. Prestressed stays. |
| | 12.2 | 14.3 | 0.346 | | Backfilling to full height. Stays disconnected. |
| | 11.5 | 14.5 | 0.333 | | Do. Readings after 7 days. |
| | 11.4 | 11.7 | | | Do. Readings after 17 days. |
| | | | 0.346 | — 0.10 | Wall movement, immediately. |
| | | | 0.333 | — 0.25 | Wall movement after 7 days. |
| | | | | — 0.35 | Wall movement after 17 days. |

## Appendix 2

*Earth pressure from pebbles*

Test No. 1

$\gamma = 1.58$ t/m$^3$
$E = K_E \cdot \gamma \cdot h^2/2$

| Number of days | Temperature in rock shelter $t_y$ °C | Temperature around pressure cell $t_i$ °C | $f_o$ 0.01 mm | $f_m$ 0.01 mm | $f_u$ 0.01 mm | $K_E$ | $\frac{f_{av}}{h} \cdot 10^4$ | Notes |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.6 | 18.1 | 0 | 0 | 0 | 0 | 0 | No backfilling. |
| 2 | 8.6 | 17.8 | 108 | 74 | 40 | 0.264 | 3.7 | Backfilling to full height. |
| 5 | 9.0 | 17.9 | 109 | 75 | 43 | 0.26 | 3.8 | Backfilling to full height. |
| 9 | 8.1 | 17.9 | 16 | 12 | 7 | 0 | 0.6 | No backfilling. |

*Earth pressure from pebbles*

Wall initially loaded by prestressed stays.

Test No. 2

| Number of days | Temperature in rock shelter $t_y$ °C | Temperature around pressure cell $t_i$ °C | $f_o$ 0.01 mm | $f_m$ 0.01 mm | $f_u$ 0.01 mm | $K_E$ | $\frac{f_{av}}{h} \cdot 10^4$ | Notes |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.3 | 17.9 | 0 | 0 | 0 | 0 | 0 | No backfilling. |
| | 7.3 | 17.9 | 110 | 71 | 40 | | | Do. Prestressed stays. |
| 2 | 8.1 | 17.8 | 114 | 78 | 48 | 0.292 | | Backfilling to full height. Stays disconnected. |
| 8 | 7.8 | 17.8 | 112 | 78 | 48 | 0.29 | | Backfilling to full height. Stays disconnected. |
| | | | 4 | 7 | 8 | | 0.3 | Wall movement after 2 days. |
| | | | 2.4 | 7 | 8 | | 0.3 | Wall movement after 8 days. |

*Earth pressure from pebbles*

Wall initially loaded by prestressed stays.

Test No. 3

| Number of days | Temperature in rock shelter $t_y$ °C | Temperature around pressure cell $t_i$ °C | $f_o$ (0.01 mm) | $f_m$ (0.01 mm) | $f_u$ (0.01 mm) | $K_E$ | $\frac{f_{av}}{h} \cdot 10^4$ | Notes |
|---|---|---|---|---|---|---|---|---|
| 0 | 7.7 | 17.7 | 0 | 0 | 0 | 0 | 0 | No backfilling. |
| | 7.9 | 17.8 | 117 | 80 | 50 | | | Do. Prestressed stays. |
| 2 | 7.8 | 17.8 | 115 | 81 | 47 | 0.31 | | Backfilling to full height. Stays disconnected. |
| | | | — 1 | 1 | — 3 | 0.31 | — 0.05 | Wall movement after 2 days. |

*Earth pressure from pebbles*

Wall initially loaded by prestressed stays.

Test No. 4

| Number of days | Temperature in rock shelter $t_y$ °C | Temperature around pressure cell $t_i$ °C | $f_o$ (0.01 mm) | $f_m$ (0.01 mm) | $f_u$ (0.01 mm) | $K_E$ | $\frac{f_{av}}{h} \cdot 10^4$ | Notes |
|---|---|---|---|---|---|---|---|---|
| 0 | 7.7 | 17.7 | 0 | 0 | 0 | 0 | 0 | No backfilling. |
| | 7.9 | 17.8 | 117 | 80 | 50 | | | Do. Prestressed stays. |
| 2 | 7.8 | 17.8 | 115 | 82 | 47 | 0.31 | | Backfilling to full height. Stays disconnected. |
| 13 | 7.5 | 17.7 | 112 | 80 | 46 | 0.29 | | Backfilling to full height. Stays disconnected. |
| 18 | 7.0 | 17.7 | 111 | 80 | 45 | 0.29 | | Backfilling to full height. Stays disconnected. |
| | | | — 1 | 1 | — 3 | 0.306 | — 0.05 | Wall movement after 2 days. |
| | | | — 6 | 0 | — 4 | | — 0.15 | Wall movement after 13 days. |
| | | | — 7 | 0 | — 5 | | — 0.20 | Wall movement after 18 days. |

*Earth pressure from pebbles*

Wall initially loaded by prestressed stays.

Test No. 5

| Number of days | Temperature: in rock shelter $t_y$ °C | Temperature: around pressure cell $t_i$ °C | $f_o$ | $f_m$ | $f_u$ | $K_E$ | $\frac{f_{av}}{h} \cdot 10^4$ | Notes |
|---|---|---|---|---|---|---|---|---|
| | | | 0.01 mm | | | | | |
| 0 | 4.5 | 17.8 | 0 | 0 | 0 | 0 | 0 | No backfilling. |
| 3 | 5.5 | 17.7 | 102 | 71 | 41 | 0.250 | 3.6 | Backfilling to full height. |
| 7 | 5.9 | 17.7 | 102 | 73 | 42 | 0.26 | 3.6 | Backfilling to full height. |

*Earth pressure from pebbles*

Test No. 6

| $f_o$ | $f_m$ | $f_u$ | $f_{av}$ | $K_E$ | $f_{av}$* red. | $\frac{f_{av,\ red.}}{h} \cdot 10^4$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0.432 | — 72 | — 3.6 |
| 84 | 52 | 19 | 52 | 0.338 | — 20 | — 1.0 |
| | | | | 0.300 | 0 | 0 |
| 162 | 104 | 39 | 102 | 0.252 | 30 | 1.5 |
| 254 | 167 | 71 | 164 | 0.203 | 92 | 4.6 |
| 345 | 228 | 110 | 228 | 0.189 | 156 | 7.8 |
| 435 | 289 | 138 | 287 | 0.186 | 215 | 10.8 |
| 501 | 356 | 171 | 343 | 0.182 | 271 | 13.6 |
| 534 | 380 | 183 | 366 | 0.180 | 294 | 14.7 |

$K_{\frac{f}{h}=0} = 0.300 \cdot 10^{-4}$

* by means of graphical interpolation, Fig. 21

*Earth pressure from pebbles*

Test No. 7 (No. 6 repeated)

| $f_o$ | $f_m$ | $f_u$ | $f_{av}$ | $K_E$ | $f_{av}$ red. | $\frac{f_{av,\ red.}}{h} \cdot 10^4$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0.329 | — 16 | — 0.8 |
| | | | | 0.300 | 0 | 0 |
| 134 | 89 | 46 | 90 | 0.208 | 74 | 3.7 |
| 264 | 202 | 105 | 190 | 0.191 | 184 | 9.2 |
| 391 | 291 | 150 | 277 | 0.191 | 261 | 13.1 |
| 517 | 377 | 196 | 363 | 0.184 | 347 | 17.3 |
| 684 | 490 | 244 | 473 | 0.184 | 457 | 22.9 |

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# Measurements in Grain Silos

**during Filling and Emptying**

**by *Werner Bergau***

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## Preface

Since the performance of measurements on silos is rather intricate case records have been called for. This report contains details of two series of measurements made 1951 and 1952 by the Mechanical Department of the Institute. Dr Werner Bergau was in charge of the tests and he has also prepared this report. An appendix has been written by Mr Torsten Kallstenius, head of the Department.

Stockholm, August, 1959

SWEDISH GEOTECHNICAL INSTITUTE

## Contents

## Synopsis

In view of recent discussions on silo pressure, results from grain silo measurements in 1951 and 1952 are presented. The measurements were carried out on a concrete silo by means of a hydraulic pressure cell and on a riveted steel plate silo by electrical wire strain gauges. The tests show that when emptying a silo wall pressure and strains are higher than when filling.

## 1. Introduction

On several occasions damage to silos has led to discussions as to what extent the silo formulas, generally based on Janssen's equation (JANSSEN, 1895), can be applied when designing silos. Even if Janssen had carried out his experiments on models it had been realised prior to this that more valuable knowledge would be gained by carrying out measurements on full-scale silos.

For the purposes of this report reference will be made only to such earlier investigations which consider not only the pressure conditions when filling and when the silo is filled but also the pressures arising when emptying. HOFFMANN (1916), who described Janssen's model tests in detail, has pointed out PRANTE (1896) as the person who made the first measurements on full-scale silos. These showed a lower wall pressure during filling than had been computed from Janssen's formula. The most noticeable factor, however, was that pressure rises during emptying were five times as great as those recorded when filling. Hoffmann commented that no one else had ever observed such pressure increases.

Subsequent investigations (BOVEY, 1904; LUFFT, 1904, see Hoffmann, p. 186) led to the conclusion that Janssen's formula, as supplemented by KÖNEN in 1896 (see Hoffmann, p. 202), could, provided that the component coefficients were suitably chosen, be considered to indicate the wall pressure when filling. In most cases only moderate increases were noted when emptying although in one case (Lufft) the emptying pressure was double the filling pressure.

Since, gradually, further progress has been made into the technique of measuring pressure exerted by granular masses (see, *e.g.*, KALLSTENIUS & BERGAU, 1956) it seems now, to some extent at least, as if the above variations were due to relatively primitive measuring methods. It had not been previously appreciated that movements of the measuring organ greatly influenced the results of such measurements.

A few years ago it was still considered that the problem of measuring pressure in silos had not been solved satisfactorily, and large-scale investigations were initiated during the 1940s in connection with the construction of silos in France and Algeria. The results of these investigations have now been published (REIMBERT, 1954; CAQUOT, 1957; DESPEYROUX, 1958). In brief, the findings were that, when filling, the wall pressures were almost in accordance with the Janssen–Könen formula or with Caquot's modified formula whereas, on the other hand, as Despeyroux says, the pressure increases when emptying still seem to be "assez capricieuses".

Reimbert carried out a series of measurements on metal silos using strain gauges and found during emptying that pressure increases up to 2.4 times the corresponding filling pressure arose. However, there were considerable variations in the results obtained for one and the same silo. The relation between the wall pressure when emptying and the pressure when filling was called the "dynamic coefficient" and Reimbert said that certain silos in France had been calculated on the basis of a dynamic coefficient of 2.

Despeyroux suggested that the uncertainty when calculating the stresses which arise when emptying a silo was probably due to several factors, *e.g.*, rate of emptying, emptying process and the properties of the filling. Further investigation was called for.

It is in this light that a report of former silo tests of the Swedish Geotechnical Institute is considered to be justified.

## 2. Measurements of the Institute

### 2 a. General

In connection with an inquiry in 1950 regarding a damaged silo the Institute was called upon to measure the wall pressure in a grain silo. At that time experiments with a hydraulic earth pressure cell (KALLSTENIUS & BERGAU, 1956) were carried out. This cell could also be used for the actual purpose. The aim was primarily to check the validity of the Janssen–Könen formula. To begin with only the pressure arising when filling was to be measured. However, when a rise in pressure during emptying was noticed during the first tests it was decided to extend the scope of the investigation to include also pressure on emptying.

Two types of test were carried out, the first, *Test Arrangement I*, on a reinforced concrete silo by direct measurements with the Institute's hydraulic earth pressure cell, the second, *Test Arrangement II*, on a circular steel plate silo by measuring strains in the steel with electrical wire strain gauges.

The present report gives an account of the measurement methods used and of the values recorded with a view to providing a better understanding of the problems associated with silo pressures.

## 2 b. Measurements on Concrete Silo

### 2 b 1. *Measurement Arrangements*

In the case of *Test Arrangement I* the earth pressure cell was used in a concrete silo owned by Uppsala Ångkvarn. The tests were carried out in January and August 1951.

Fig. 1 shows a schematic drawing of the concrete silo in question. The diameter was 6.3 metres and the height of the cylindrical section 22.5 metres. The pressure cell was placed in a hole chopped out of the silo wall so that the surface of the cell lying against the grain was flush with the surface of the inner wall (Fig. 2 a).

The grain—wheat—was poured into the silo through an opening at the top. The grain was weighed on automatic scales. Owing to the filling opening being offset from the centre of the silo the grain surface was irregular and, in the case of the January tests, the surface was levelled by hand before the pressure measurements were taken. The height of this horizontal was measured with a plumbline from the centre of the top of the silo.

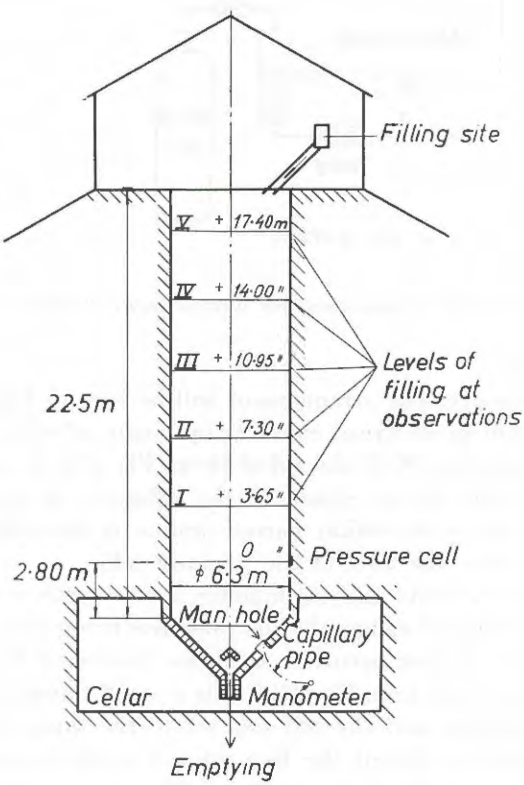


*Fig. 1. Arrangement for measurement of wall pressure in concrete silo*

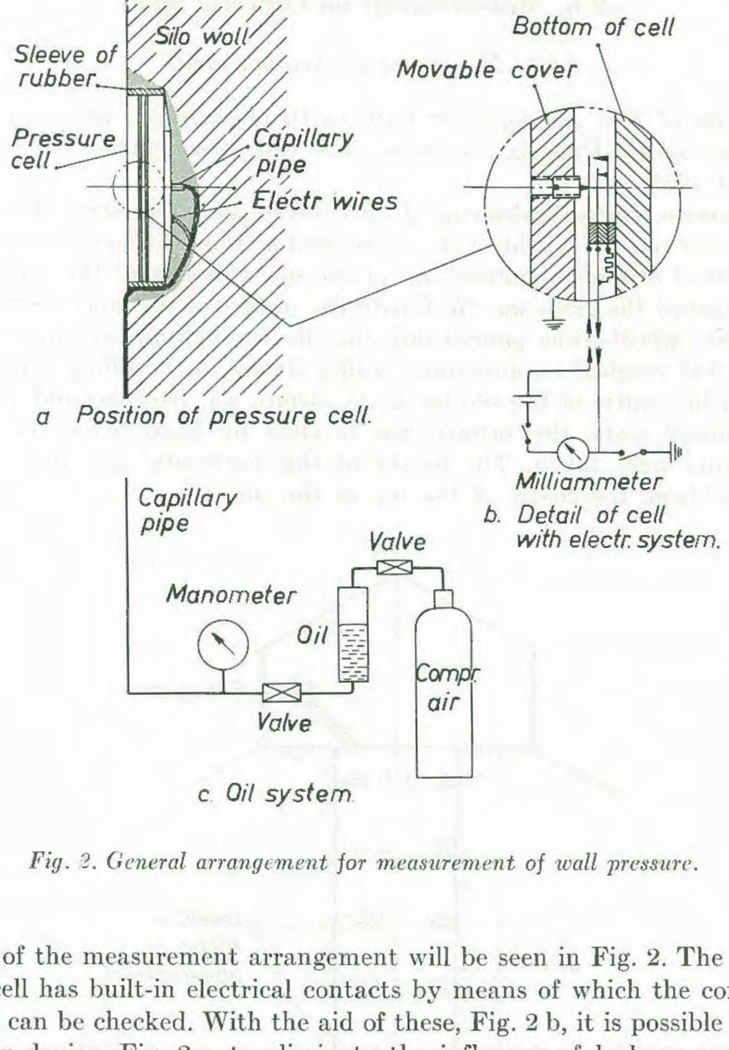


*Fig. 2. General arrangement for measurement of wall pressure.*

Details of the measurement arrangement will be seen in Fig. 2. The hydraulic pressure cell has built-in electrical contacts by means of which the compression of the cell can be checked. With the aid of these, Fig. 2 b, it is possible by means of a filling device, Fig. 2 c, to eliminate the influence of leakage and to keep the travel of the cell cover within narrow limits. A Bourdon manometer was placed 6.2 metres below the level of the pressure cell.

During the January tests the manometer used together with the cell had a rather large measurement range, the silo pressures being only about 0.3 kg/cm$^2$. In addition, a slight oil leak occurred and was discovered by checking the position of the electrical contacts. To offset this a small quantity of oil was added before the measurements and the cell was set to the same zero value. In view of this it was decided to repeat the test using a revised measurement system.

The manometer was replaced by another with a more suitable measurement range and the whole system was calibrated under similar conditions to those prevailing in practice and carefully checked to ensure that there were no leaks. The measurement arrangement was then built into the silo wall in the same way as had been done during the January tests. The measurement system was free from leakage after being installed and on no occasion was it necessary to refill the cell. Thus there was no forced movement of the cell cover against the grain.

### 2 b 2. *Performance of the Tests with the Concrete Silo*

The final test with the cell in the concrete silo was carried out in August 1951. The unit weight $\gamma$ and the water content $w$ of the grain were determined in the mill laboratory immediately before the silo was filled ($\gamma = 0.84$ kg/dm$^3$,

*Table 1*

| Date | | Level | Height of fill above cell: measured with plumbline m | Height of fill above cell: corresp. plane m | Quantity put in metric tons | Temp. (cellar) °C | Manometer readings kg/cm$^2$ | Wall pressure (from calibration curve) kg/cm$^2$ |
|---|---|---|---|---|---|---|---|---|
| 1951 | | | | | | | | |
| 9/8 | $9^h$ | 0 | 0 | 0 | 0 | 17.4 | 0.495 | 0 |
| | $12^h$ * | I | 3.45 | 3.65 | 100 | 17.4 | 0.635 | |
| | $15^h$ | | 3.65 | 3.65 | | 17.4 | 0.625 | |
| 10/8 | $9^h$ | | 3.65 | 3.65 | | 17.3 | 0.615 | 0.11 |
| | $14^h$ | II | 7.30 | 7.30 | 100 | 17.3 | 0.760 | |
| 11/8 | $9^h$ | | 7.30 | 7.30 | | 16.9 | 0.755 | 0.20 |
| | $12^h$ | III | 11.15 | 10.95 | 100.3 | 17.1 | 0.860 | |
| 13/8 | $9^h$ | | 11.15 | 10.95 | | 16.8 | 0.885 | 0.26 |
| | $12^h$ | IV | 14.30 | 14.00 | 83 | 16.8 | 0.935 | |
| 14/8 | $9^h$ | | 14.30 | 14.00 | | 16.9 | 0.965 | 0.30 |
| | $12^h$ | V | 18.35 | 17.40 | 91.7 | 16.9 | 1.025 | |
| 15/8 | $9^h$ | | 18.35 | 17.40 | | 16.5 | 1.035 | |
| 16/8 | $9^h$ * | | 18.35 | 17.40 | | 16.3 | 1.035 | |
| | $12^h$ | | 17.40 | 17.40 | | 16.4 | 1.030 | |
| 17/8 | $9^h$ | | 17.40 | 17.40 | | 16.2 | 1.055 | |
| 20/8 | $7^{15h}$ | | 17.40 | 17.40 | | 16.5 | 1.035 | 0.33 |

Material: wheat.
Temp. in the silo: + 18°C.

* Surface of the filling levelled.

$w = 13.5\,\%$ of dry substance). At the same time measurements on a similar type of wheat were made at the Institute laboratory. The values obtained were $\gamma = 0.83$ loosely filled and $\gamma = 0.90$ densely compacted. A check on the unit weight of the mass which is filled into the silo above zero level gives an average value of $\gamma = 0.88$, but this value is surely too high in view of the fact that the underlying layer has been compacted by the filling. Which of these values is representative for the conditions within the silo is difficult to say. On the other hand, recent tests at the Institute have shown that a certain degree of compaction occurs when a thin stream of granular material is allowed to fall freely onto a large surface.

The silo was filled by steps. The wheat was first filled up to the pressure cell and the surface levelled by hand. The level of the cell was used as zero for height measurements. 100 tons of wheat were than added and the height above the cell and the pressure recorded before and after the levelling of the surface of the grain, Table 1. The surface of the filling was not levelled after the next filling steps except after the last one. Consequently the height of the fill above the cell, measured with the plumbline, and also that of a corresponding plane (assuming the grain surface being levelled) have been indicated.

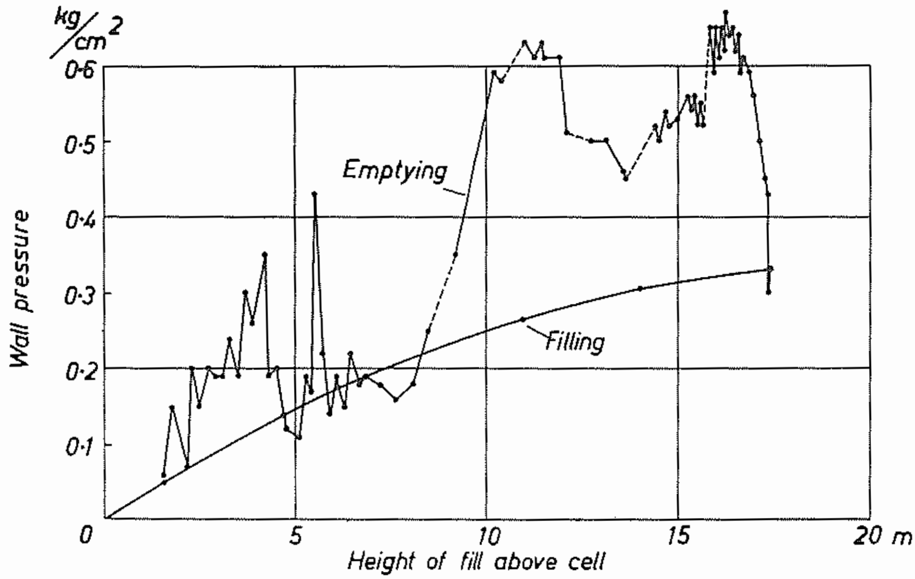


*Fig. 3. Wall pressure in the concrete silo.*

At each filling step the pressure was measured immediately after filling and immediately before next step. The latter measurement showed somewhat higher values in the higher pressure ranges than the former. This was due in part to a time-influenced lag, hysteresis, in the Bourdon manometer since a similar tendency was noted when calibrating on the same time programme as was applied during the test on the silo. The manometer in question had a measurement range of 2.5 $kg/cm^2$, and the gradation interval was 0.05 $kg/cm^2$. The values shown in Table 1 and Fig. 3 have been corrected for hysteresis and temperature variation. The corrections were only one or two per cent.

After the silo had been kept filled for three days—towards the end of which period the wall pressure tended to decrease—the silo was emptied. The manometer was observed the whole time except for certain interruptions, mainly when the level of the wheat was checked with the aid of a plumbline from the top of the silo. The weighing of the wheat removed was like all other observations time-recorded. The rate of emptying averaged 0.028 metres/minute or in quantity 0.87 $m^3$/min. and the rate of weight 0.75 tons/min., *i.e.*, an average unit weight $\gamma = 0.87$ for the mass emptied. At the same time as acoustic observations indicated that the vertical movements of the grain were uneven, the manometer showed considerable pressure variations. The side pressure obtained from the calibrations is shown in Fig. 3. As long as the manometer was under observation the measurement points have been joined with full lines. Breaks in the observations have been indicated by dashed lines.

When the results of the measurements were obtained in 1951 the question was raised as to whether the results in respect of an individual measurement point, *i.e.*, at the point where the pressure cell was located, could really be considered as being representative for the distribution of pressure over the entire level. Therefore an extension of the investigation was considered advisable.

## 2 c. Measurements on Steel Plate Silo

### 2 c 1. *Measurement Arrangements*

Supplementary tests were carried out on a steel silo, the purpose being to measure the strains at several points simultaneously with strain gauges, *Test Arrangement II.* Unfortunately, the silo on which the measurings were to be performed was not ideal but, nevertheless, the investigation is valuable in the light of future research of this nature and it may also give a qualitative confirmation of the tests made with the concrete silo.

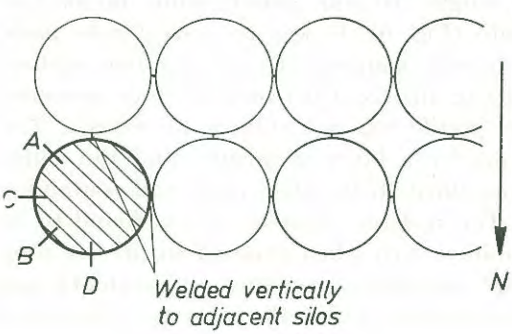


*Fig. 4. Situation of steel plate silo (A–D measurement points).*

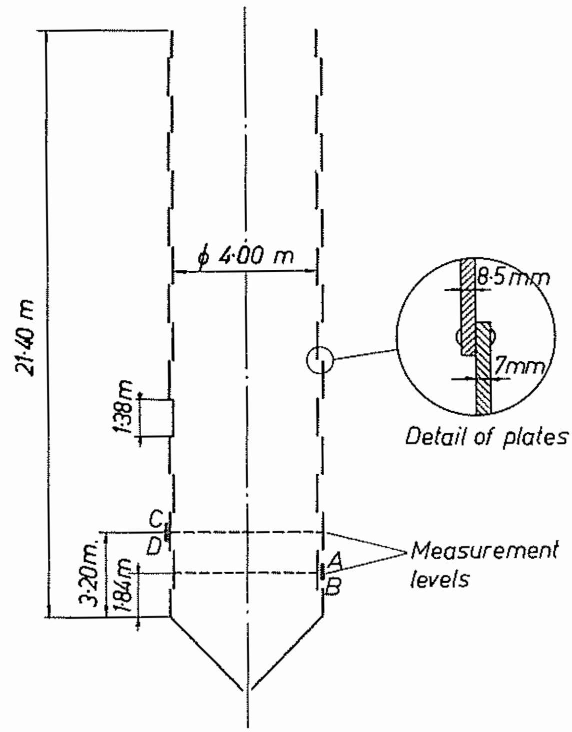


*Fig. 5. General dimensions of the steel plate silo.*

The silo in question was one unit in a double row of 20 silos at the Tre Kronors Kvarn, Stockholm (Fig. 4). The silo was made up of riveted overlapping steel plate rings. The height of the cylindrical section of the silo was 21.4 metres and the diameter 4 metres (Fig. 5). Each silo was welded to the adjacent ones and consequently the silo under examination was only accessible over three-quarters of the circumference.

In the middle of four plates (two belonging to an inner ring and two to an outer) strain gauges (type Gustafsson, length 100 mm, 2000 $\Omega$, gauge factor 2.49) were affixed vertically and horizontally (Fig. 6). It was not possible to place gauges inside the silo. The passive dummy gauges, intended for temperature compensation, were placed on an angle profile fixed between the four measurements points in such a way that the profile was not subject to stresses. The gauges were protected against moisture by a layer of lacquer and the gauge groups were covered by rubber sheaths glued to the steel plate and containing silica gel as the moisture absorbant. The moisture protection was found to be reliable over the course of several months, even when exposed to driving rain.

Every gauge was checked as regards insulation resistance and stability and more than half of the number used were replaced before the gauges were accepted

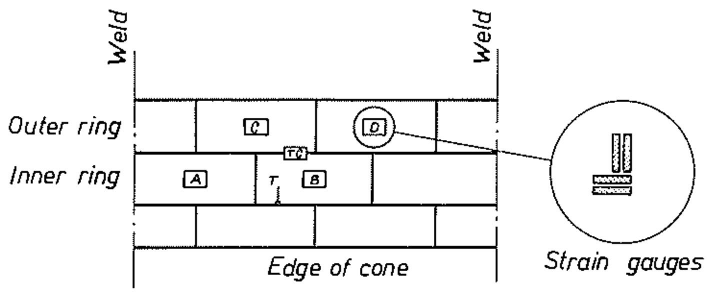


*Fig. 6. Arrangement of measurement points at the steel plate silo (A–D measurement points; TC gauges for temperature compensation).*

as reliable. Changes in the zero readings were observed between the tests. Whether these were due to creep of the gauges or stress changes in the steel plates could not be established since the strains in the empty silo were affected both by temperature and stress changes in the adjacent silos. Zero travel due to creep had probably not influenced the test results since the measuring time was short. To some extent the zero travel was checked by making comparative measurements on the unloaded passive gauges. In the case of the third test all the gauges were doubled to reduce the influence of gauge errors.

Changes in the gauge resistance were measured with the aid of a DC bridge (type Gustafsson) which enables measurements regardless of the wire resistance in that double wires are led to each gauge contact. A light spot galvanometer (type Norma) was used as the zeroing instrument.

The temperature of the steel plates of the silo was determined with a mercury thermometer, the lower part of which was in contact with the steel and insulated from the air.

### 2 c 2. *Performance of the Tests with the Steel Plate Silo*

The tests were carried out between July and September 1952. Both filling and emptying proceeded continuously and measurements were taken throughout these processes. The level of the material in the silo was checked by measurements with a plumbline and the quantity of the mass by time-recording automatic scales. Strain measurements were made in such a way that the gauges were successively connected to the DC bridge. In Tests Nos. 1 and 2, a rapid-action switch was used, and the time interval between the measurement series was 5–15 minutes. In Test No. 3 the wires were connected direct to the bridge and the interval was 10–15 minutes. Thus the measurements at the single points were not continuous.

The unit weight and water content of the grain were determined in the mill laboratory. Much higher values will be found when computing the unit weight from filling and emptying rate values in Table 2.

Three tests were made, in accordance with the following table.

*Table 2*

| Filling data | Test 1 | Test 2 | Test 3 |
|---|---|---|---|
| Date | 11.7.1952 | 15.7.1952 | 3.9.1952 |
| Material | Barley | Wheat | Wheat |
| Unit weight, kg/dm$^3$ (Lab. values) | 0.754 | 0.788 | 0.805 |
| Water content, % of dry substance | 14.3 | 14.9 | 14.9 |
| Quantity, metric tons | 200 | 194.8 | 205 |
| Height of fill above cone edge, metres | 17.8 | 16.5 | 17.6 |
| Filling rate, metres/min | 0.139 | 0.127 | 0.110 |
| , metric tons/min | 1.53 | 1.48 | 1.26 |
| Emptying rate, metres/min | 0.149 | 0.135 | 0.118 |
| , metric tons/min | 1.63 | 1.57 | 1.34 |
| Unit weight, computed from filling and emptying rate values | 0.871<br>0.870 | 0.929<br>0.929 | 0.908<br>0.906 |

Figs. 7 a–d show the measurement results for every measurement point for each of the three tests. The value U is the difference between the reading on the bridge before the test and the actual reading. By multiplying by the constant $1.073 \cdot 10^{-6}$ (which constant contains both the bridge factor and the gauge factor) the strain $\varepsilon$ is obtained.

The fact that the measurement values after Test No. 3 did not return to zero is probably due to stresses in the silo caused by adjacent silos being filled at the same time. It was not possible to avoid these disturbances since Test No. 3 was made during the peak season. When tests Nos. 1 and 2 were carried out, the adjacent silos were empty and the deviations were considerably less. The remaining deviation is certainly due to temperature strains. The rise in temperature from the time when the test was started in the morning to its end in the afternoon was, on the average, 4° C.

When comparing the results, it should be borne in mind that different materials were used (see Table 2). In general it can be said that when filling the strain values are less at the outer ring than at the inner. When emptying, the inner ring shows increased horizontal strain. The transition from horizontal strain to compression which the outer ring then shows indicates that bending stresses have occurred. Unfortunately these bendings prevent a calculation, free from objections, of the pressure from inside the grains.

However, there is no doubt but that the silo has been subjected to higher stresses at certain parts during the emptying process than during filling. MEHMEL (1956), referring to a coal silo, has shown that stress concentrations may be dangerous. According to his calculations based on the deformation of vertical girders the pressure when emptying was 2.37 times the filling pressure.

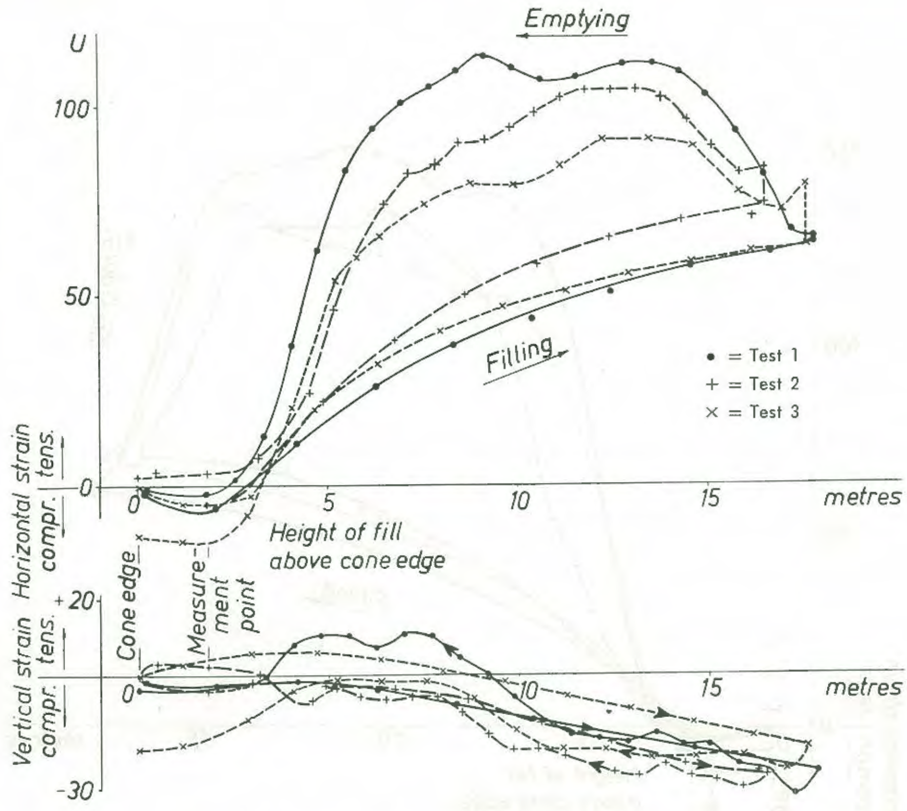


*Fig. 7 a. Results of strain measurements on the inner ring in point A.*

The measurements also indicate that caution must be taken when the pressure distribution in a steel plate silo is determined from only few measurement points. In the case in question, for example, an estimation of the pressure distribution either on the inner or on the outer ring would have led to different conclnsions. This also explains why contradictory information may be found in the literature.

## 3. Viewpoints Concerning the Emptying Mechanism in a Silo

It seems well-founded to say that the task of investigating the effect of emptying should be initiated by an attempt to understand the nature of the movement of the mass. When one stands on the surface of the grain mass in a silo, which is being emptied via a centre discharge tube, a slight downwards movement is felt close to the wall but, a short distance away, the movement is more of an inward direction.

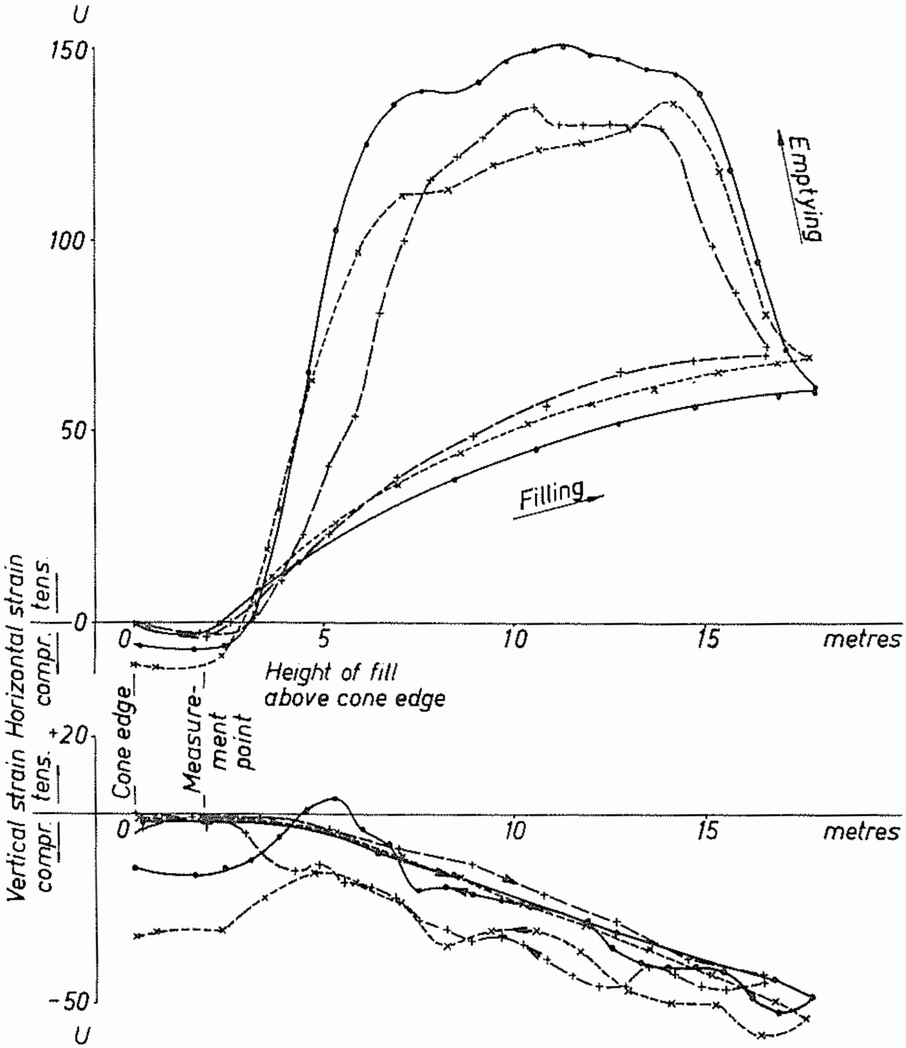


*Fig. 7 b. Results of strain measurements on the inner ring in point B.*

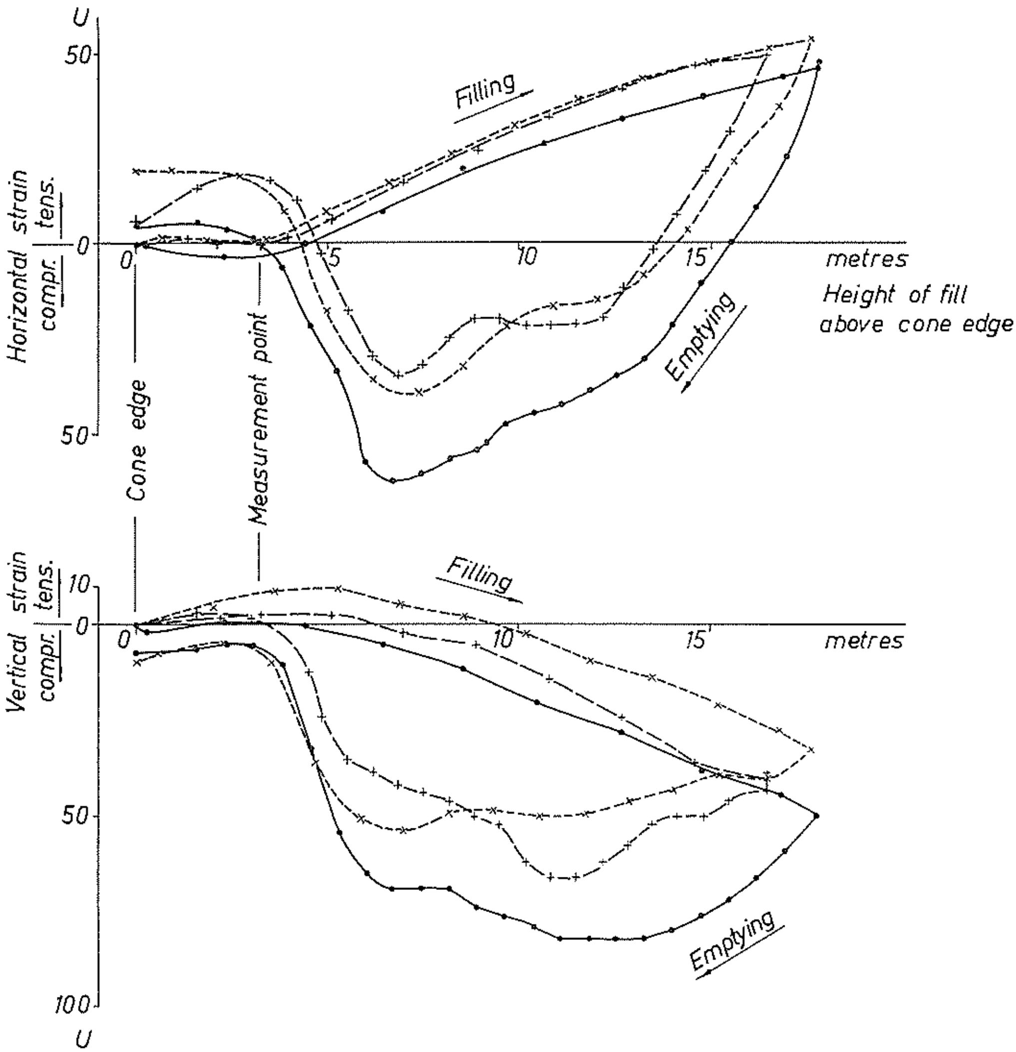


*Fig. 7 c. Results of strain measurements on the outer ring in point C.*

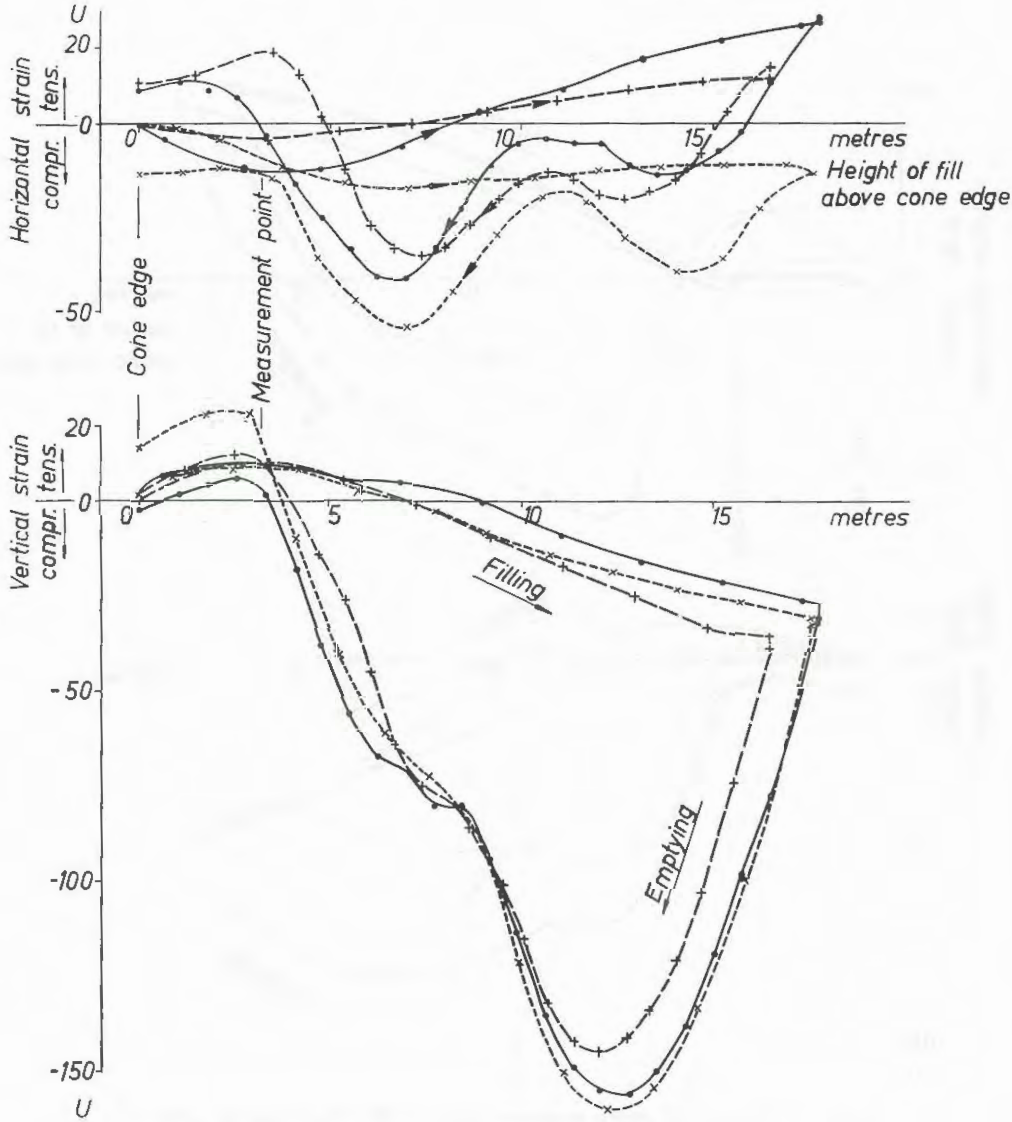


*Fig. 7 d. Results of strain measurements on the outer ring in point D.*

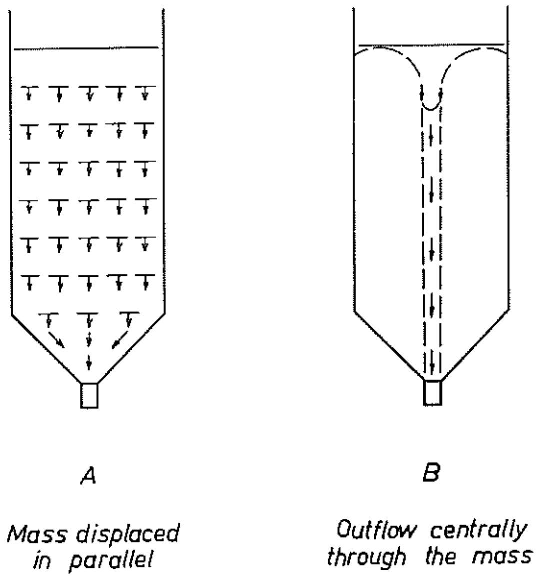


*Fig. 8. Movement in the mass when emptying a silo (hypothetic cases).*

It may be further noted that, when emptying, the discharge in the middle of the upper surface is not always even. It appears at times as if the vertical centre movement had ceased altogether. It is probable that arching then takes place in the mass. As discharge proceeds the archings will collapse. These collapses are followed by marked "shocks" in the mass which can be distinctly heard. These sounds were heard on several occasions in the concrete silo at Uppsala but seldom in the steel plate silo at Stockholm.

One can imagine two hypothetical extreme cases as to the way in which a granular mass could move downwards when a silo is emptied via a central discharge shaft (Fig. 8), *viz.*, A: the whole mass moves downwards in parallel to the vicinity of the bottom cone where the travel is concentrated by the discharge tube, and B: the outer part of the mass adheres to the walls and emptying is going on from above as through a pipe passing through the material.

In reality it can be thought that both cases are dominated by wall friction. In case A, where the wall friction is insufficient to prevent the travel, it should be possible to assume that the wall pressure—if friction is neglected—was equal to pressure at rest. Such a pressure is suggested by, for instance, Wästlund (1939). In case B the wall friction is sufficient to prevent movement at the wall and the movement of the mass is in the centre where forces arise producing horizontal supplimentary forces on the wall, especially in the case of compacted dilatant material.

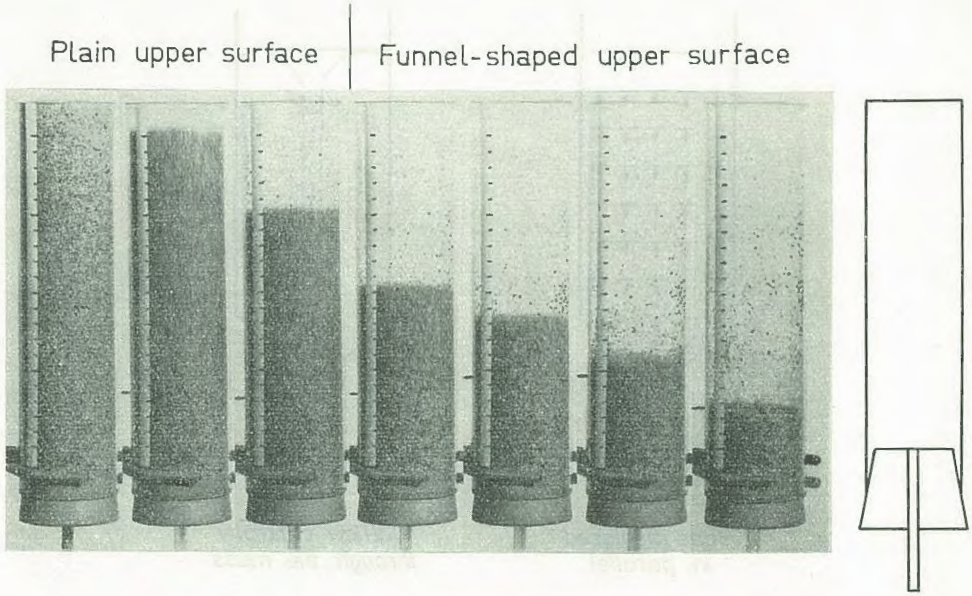


*Fig. 9. Outflow test with coarse sand. (The boundary of the flowing part is marked to the left of the tubes. Glass tube diameter 61 mm.)*

These two extreme cases can be demonstrated by making an experiment with a glass tube filled with coarse sand (see Fig. 9). Here we find case A at the top of the tube, then after a certain amount of material has poured out, a combination of Case A and Case B with Case B dominating towards the end of the process. A still better demonstration is afforded by a similar experiment (Fig. 10). Here a ball indicates the shape of the upper surface, and we can see how long Case A exists and how slowly in this case the funnel (which is an indication of Case B) is formed. The ball is then sucked into the funnel, travels towards the bottom of the funnel and, finally, blocks the outlet.

KVAPIL (1955) has made an exhaustive study of the discharge of granular and lumpy masses by means of models. He arrives at the conclusion that there is a primary motion which is vertical in character and which tends to break up the grain contacts vertically. Furthermore there is a secondary motion where the grains turn round and change their position in a horizontal direction with the effect of vertical swelling of the material. Both volumes of movement form elipsoids, overlapping each other around the same main axis, which is the axis of the outlet.

Some other studies showing certain effects of importance in this connection have been carried out at the Institute (see Appendix).

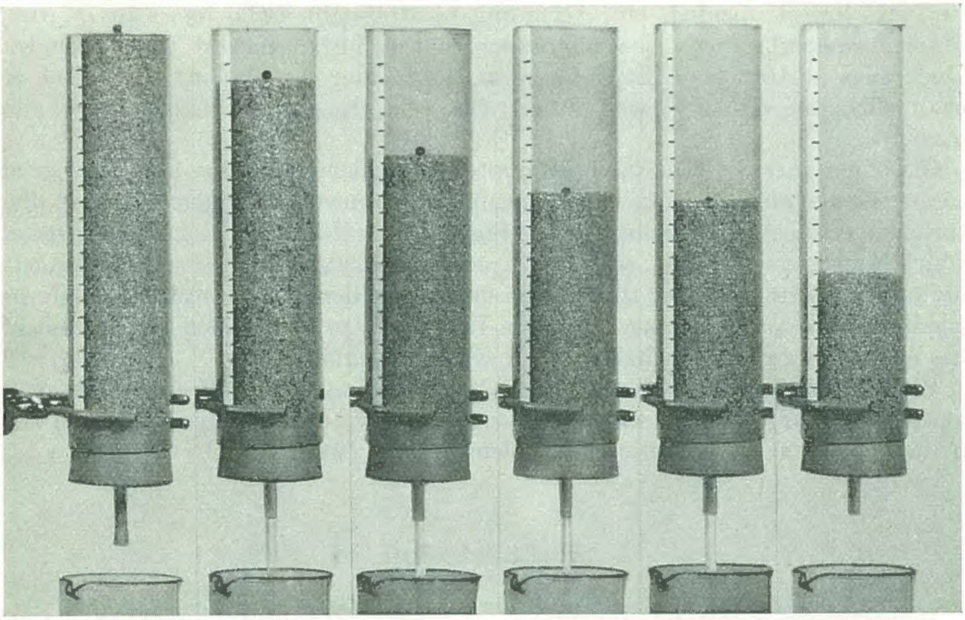

*Fig. 10. Sand pouring out from a glass tube. (The ball on the top of the sand surface indicates the shape of the upper surface.)*

## 4. Measures to Avoid Pressure Build-Up on Emptying

It may be of interest to examine some of the practical measures made to prevent pressure build-ups on the silo walls during the emptying process.

Caquot, dealing with the emptying process, is of the opinion that very unfavourable and indefinable pressures arise when the silo is emptied via a central cone. If, on the other hand, discharge is via openings spread around the whole circumference, a state of equilibrium arises.

Reimbert suggests that a pipe, provided with holes of suitable size in the pipe walls, be used *(tube dynamique)*. Emptying is performed via the top holes to begin with, while the lower holes are blocked owing to arching. Thus the entire mass remains in position, and discharge is always effected from the upper surface. Measurements made during the course of this emptying process showed that the pressure on emptying did not exceed that arising during the filling process. A similar arrangement was used by Miersch (see Hoffmann, p. 171) on silos at Frankfurt-am-Main at the beginning of the century. In this case the central tube consisted of short pieces of tube so connected as to leave gaps between the individual components. In this case, too, discharge is from above via the uppermost opening. After this has been passed the grain is discharged through the next lowest opening, *etc.* According to Hoffmann (who refers to DUHLE, Massentransport) there is a forerunner to this installation at the Alexandra Dock silos at Liverpool. In the case of these older installations the object of channelling the discharge was rather to prevent the general disturbance of the mass.

Other designers of silos have attempted to achieve the same aim by special arrangements which caused the mass to move downwards without any local disturbance (Case A). An example is the Sinclair silo (Hoffmann, p. 43). The system, which was known at the end of the 18th century, embodies several funnels uniformly distributed over the bottom surface and designed to ensure the uniform sinking of the mass. HUART (1855, see Hoffmann, p. 47) divided the bottom of the silo into two funnels fitted with a system of guide rails.

Working with models, Kvapil has also investigated the effect of half-open vertical discharge tubes and also of vertical guide rods and found that, in the main, the grains moved vertically downwards in parallel.

## 5. Conclusion

In conclusion it may be said that:

Both our tests and later information seem to indicate that, during the emptying process, the silo walls can be exposed to pressure increases. These can be in excess of the 10–20 per cent referred to in earlier literature.

By using new measurement methods the stresses and pressure can be determined more accurately without the measuring instrument influencing the distribution of pressure.

In the case of composite silos (riveted silos placed in rows) the measurements must be carried out at many points suitably distributed.

Measurements with pressure cells show the silo pressure, while strain and stress measurements indicate the stresses in the construction and consequently both types of measurement should be carried out simultaneously wherever possible.

It would therefore appear important that more detailed studies be made of the emptying process. These studies should be combined with investigations about the shape and the location of the outflow and their effect on the stress distribution in the mass so that suitable steps can be taken to prevent the build-up of dangerous and indeterminable excess pressure.

# Appendix

## A Study of the Flow Pattern when Emptying a Vessel

by ***Torsten Kallstenius***

The analysis of the stresses arising when emptying a silo through a hole at the bottom is facilitated by a study of the flow of the mass as has been made by, *e.g.*, Kvapil in his extensive work.

During the first part of 1955 some tests were also performed by the author with the aim of studying the volume involved when piping occurs in an ungraded filter (*cf.* W. Kjellman, Unorthodox Thoughts about Filter Criteria, Stockholm 1955—presented at the Soil Mechanics Conference in Ljubljana 1955). The results seem to give additional experience to that given by Kvapil and are therefore mentioned here.

A vessel of half-moon shaped cross-section (radius 250 mm and height 500 mm) had a vertical wall of lucite. At the bottom of the vessel an opening was provided, also with half-moon shaped section and with its centre close to the lucite-wall. The opening was provided with trap-door shutter. Three openings were used with radii 8 mm, 25 mm and 50 mm, respectively.

The vessel was filled with normal sand in horizontal layers of 1 cm thickness, lightly tampered to a void ratio estimated to be about 0.70. Two types of sand were used, one of greyish colour and one of yellowish colour. When photographing the sand through the lucite-wall, using orthochromatic plates and a pale blue filter, the layers could be clearly distinguished.

By opening and shutting the trap-door a certain amount of sand could be emptied in a measuring container and the weight and volume of the emptied mass could be determined (about $0.1 + 0.2 + 0.5 + 1.0 + 2.0 + 5.0$ litres were emptied).

Fig. 11 shows pictures from a test using the smallest opening ($r = 8$ mm). As can be seen in Fig. 11 a, the sandlayers were not perfectly plane before the emptying was started; they were curved near the periphery of the vessel in particular. Fig. 11 a shows the deformations after $\approx 0.1$ l had been emptied. With increased emptied volume the "bulb" of disturbance increases its diameter and height. When it reaches the top of the sand-fill a funnel is formed. If additional sand is filled into this funnel the disturbed zone increases in diameter through erosion (Fig. 11 b–11 d).

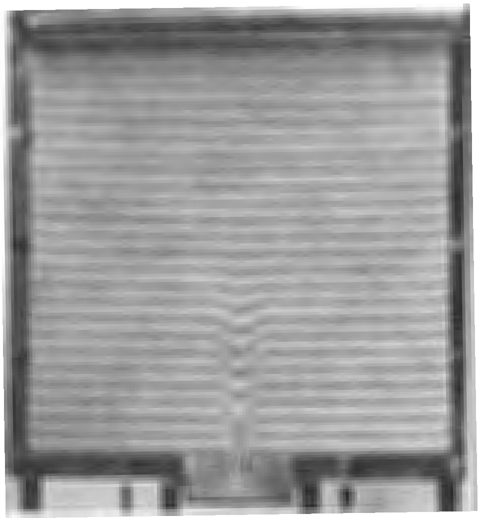

*Fig. 11 a.*

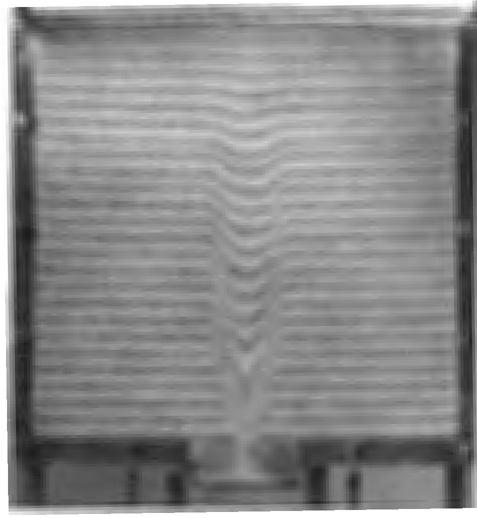

*Fig. 11 b.*

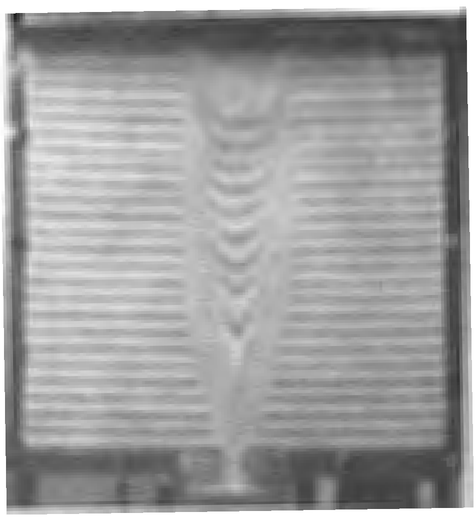

*Fig. 11 c.*

*Fig. 11 d.*

The shape of the bulb is dependent on the packing of the sand. Fig. 12 shows another test at a phase corresponding most to Fig. 11 b and it is evident that the diameter-height ratio of the bulb is variable as well as the steepness of the gradients of shear deformation in the zone intermediate between flowing and remaining material.

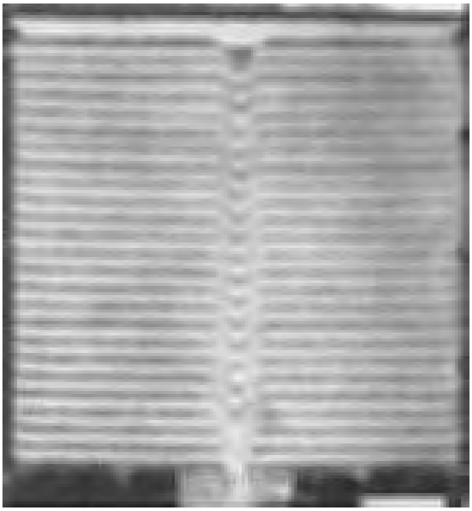

Fig. 12.

*Figs. 11–13. Flow patterns when emptying a vessel.*

*Figs. 11–12. Vessel opening $r = 8$ mm.*

*Fig. 13. Vessel opening $r = 50$ mm.*

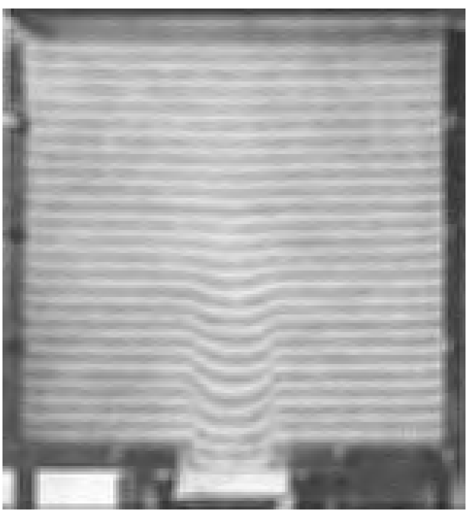

*Fig. 13 a.*

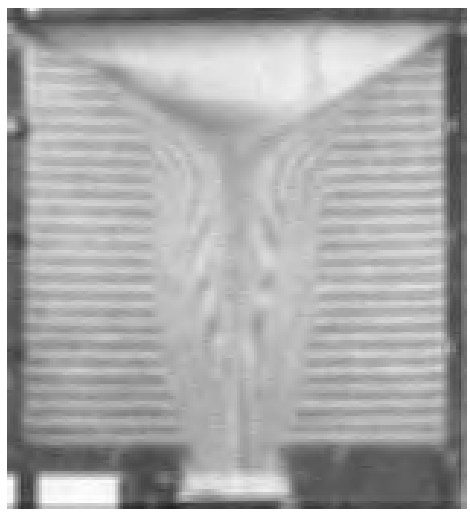

*Fig. 13 b.*

Fig. 13 a–b show the corresponding flow patterns when using an opening with radius 50 mm. It will be observed that the size of the opening has a certain influence.

By studying the curvature of the surfaces in the bulbs, and by measuring the thickness of the sand layers in the different parts of the pictures, one is able to determine the behaviour of the sand in the different parts of the vessel. The pictures have been used for such purposes in many cases in our research. The intention has been to repeat the tests with more careful determination of densities, weights and volumes than was done in the above preliminary tests but this has not yet been done, and we take the opportunity to present some of the pictures in connection with the report of the silo measurements.

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# LIST OF PROCEEDINGS OF THE SWEDISH GEOTECHNICAL INSTITUTE

| No. | Title | Year | Price Sw. Crs. |
|---|---|---|---|
| 1. | Soil Sampler with Metal Foils. Device for Taking Undisturbed Samples of Very Great Length. *W. Kjellman, T. Kallstenius* and *O. Wager* | 1950 | 4:— |
| 2. | The Vane Borer. An Apparatus for Determining the Shear Strength of Clay Soils Directly in the Ground. *Lyman Cadling* and *Sten Odenstad* | 1950 | 4:— |
| 3. | Device and Procedure for Loading Tests on Piles. *W. Kjellman* and *Y. Liljedahl* | 1951 | 3:— |
| 4. | The Landslide at Sköttorp on the Lidan River, February 2, 1946. *Sten Odenstad* | 1951 | 4:— |
| 5. | The Landslide at Surte on the Göta River, September 29, 1950. *Bernt Jakobson* | 1952 | 8:— |
| 6. | A New Geotechnical Classification System. *W. Kjellman, L. Cadling* and *N. Flodin* | 1953 | 4:— |
| 7. | Some Side-Intake Soil Samplers for Sand and Gravel. *Torsten Kallstenius* | 1953 | 4:— |
| 8. | Influence of Sampler Type and Testing Method on Shear Strength of Clay Samples. *Bernt Jakobson* | 1954 | 4:— |
| 9. | Some Relations between Stress and Strain in Coarse-Grained Cohesionless Materials. *W. Kjellman* and *B. Jakobson* | 1955 | 4:— |
| 10. | Accurate Measurement of Settlements. *W. Kjellman, T. Kallstenius* and *Y. Liljedahl* | 1955 | 4:— |
| 11. | Influence of Organic Matter on Differential Thermal Analysis of Clays. *Lennart Silfverberg* | 1955 | 4:— |
| 12. | Investigations of Soil Pressure Measuring by Means of Cells. *Torsten Kallstenius* and *Werner Bergau* | 1956 | 4:— |
| 13. | Pore Water Pressure Measurement in Field Investigations. *Torsten Kallstenius* and *Alf Wallgren* | 1956 | 4:— |
| 14. | A New Approach to the Determination of the Shear Strength of Clay by the Fall-Cone Test. *Sven Hansbo* | 1957 | 4:— |
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| 16. | Mechanical Disturbances in Clay Samples Taken with Piston Samplers. *Torsten Kallstenius* | 1958 | 6:— |
| 17. | Measurements of the Pressures of Filling Materials against Walls.<br>Earth Pressure from Friction Soils. A Report on Half Scale Tests. *Arne Rinkert.*<br>Measurements in Grain Silos during Filling and Emptying. *Werner Bergau* | 1959 | 6:— |

## FÖRTECKNING ÖVER MEDDELANDEN FRÅN STATENS GEOTEKNISKA INSTITUT

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| No. 1. | Kortfattat kompendium i geoteknik 1946 | 1946 | utgånget |
| 2. | Redogörelse för Statens geotekniska institutets verksamhet under åren 1944—1948 | 1949 | 2:— |
| 3. | »Bra borrat — bättre byggt». Meddelande utgivet till institutets deltagande i utställningen »Bygg bättre», Nordisk Byggnadsdag V | 1950 | — |
| 4. | Anvisningar för geotekniska institutets fältundersökningar. Del 1. Undersökningarnas planläggning och omfattning. Sonderingsmetoder. Vingborrning. Provtagning. Uppgörande av borrningsritningar. *Nils Flodin* | 1959 | 8:— |
| | D:o med plastpärmar för fältbruk | | 10:— |
| 5. | Kompendium i geoteknik 1959 | 1959 | 12:— |