STATENS GEOTEKNISKA INSTITUT
SWEDISH GEOTECHNICAL INSTITUTE

RAPPORT
REPORT No 3

Methods for reducing undrained shear strength of soft clay

K.V. HELENELUND

LINKÖPING 1977

STATENS GEOTEKNISKA INSTITUT
SWEDISH GEOTECHNICAL INSTITUTE

RAPPORT
REPORT No 3

Serien "Rapport" ersätter våra tidigare serier: "Proceedings" (27 nr), "Särtryck och Preliminära rapporter" (60 nr) samt "Meddelanden" (10 nr).

Our new series "Report" supersedes the previous series: "Proceedings" (27 Nos), "Reprints and Preliminary Reports" (60 Nos) and "Meddelanden" (10 Nos).

Methods for reducing undrained shear strength of soft clay

K.V. HELENELUND

Denna rapport hänför sig till forskningsanslag 760102-3 från Statens råd för byggnadsforskning.

LINKÖPING 1977

PREFACE

This report deals with different methods for reduction of the undrained shear strength of normally consolidated and slightly overconsolidated clay according to field vane tests or fall-cone tests. In Sweden a gradual reduction of the shear strength has been made on the basis of the cone liquid limit of the clay since the 1930-ties. In recent years a number of new reduction methods have been proposed and a comparison of these methods is therefore of interest. As the shear strength reduction is connected with the risk of failure and the factor of safety to be used in geotechnical design, the report also contains a discussion of the probability of failure and the optimum factor of safety.

The author is grateful to Dr. Tech, L Andréasson, Head of the Swedish Geotechnical Institute, and to his colleagues at the Institute for fruitful discussions during his 3 month´s stay at SGI in Linköping in the fall 1976. Valuable information was also obtained from field investigations and discussions with Prof. R Pusch and his colleagues at the Department of Geotechnology of the University of Luleå. Important conclusions have been drawn from case records and observations reported by a number of persons and the author is grateful for this valuable information.

The investigations reported in this paper were sponsored partly by the Swedish Council for Building Research in Stockholm and partly by the Finnish Technical Research Council in Helsinki.

Linköping May 1977

K.V. Helenelund

CONTENTS

1 ABSTRACT

Methods for reducing undrained shear strength of soft clay

Observations from failures of slopes and embankments, and loading tests show that the shear strength measured in field vane tests and fall-cone tests in organic and plastic clay has to be reduced in order to correspond to the real shear strength of the clay. In Sweden a gradual shear strength reduction has been made on the basis of the cone liquid limit, as soon as this exceeds 80%. In recent years several new methods have been proposed for the shear strength reduction.

Some observations indicate that the reduction used today has been in certain cases insufficient. A comparison between different reduction methods and the corresponding risks of failure has therefore been made.

The risk of failure has been analysed according to the scatter of the case records used by Bjerrum for deduction of his reduction factor and also according to the variations of the ratio of vane shear strength and effective overburden pressure (τ_v/σ_o') in different types of clay.

2 SUMMARY

Different methods for reduction of the undrained shear strength of soft clays according to field vane tests and fall-cone tests are presented and compared, paying special attention to the risk of failure associated with these reduction methods. Comparisons of the probability of failure according to case records and τ_v/σ_o'-ratios indicate, that there are cases, where the probability of failure will be remarkably high, if the shear strength reduction is made using the same curve or formula, for instance eqn (11) or (14) in all types of clays.

A statistical analysis of Swedish clays indicates, that there is a difference between the probability of failure of fresh- and brackish-water clays and salt-water sedimented clays, when the same strength reduction is used for both types of clay. Relatively high τ_v/σ_o'-ratios have been observed in fresh- and brackish-water clays, but the high τ_v-values cannot be relied upon to the same extent as in marine clays, as the structure of fresh-water clays is more easily disturbed by shear displacements before rupture.

Clays with high apparent τ_v/σ_o'-ratio, resulting from e.g. secondary consolidation, may thus need a greater shear strength reduction. In such clays the reduction could be based on the τ_v/σ_o'-ratio, using eqn (12) or Fig. 7. Another possibility is to reduce the shear strength on the basis of the overconsolidation ratio (Ladd & Foott 1974).

Instead of using different reduction factors for different clays an alternative method is not to reduce the shear strength at all, but to correct the theoretical factor of safety by dividing it with an empirical factor of safety at failure (F_{sf}). The liquid limit or τ_v/σ_o'-ratio used as a basis for the correction should then be calculated as a mean value

for the critical slip surface. This method gives satisfactory results in fairly homogeneous soils, but it can lead to errors in non-homogeneous and layered soils, e.g. regarding location of the critical slip surface.

The shear strength reduction recommended by SGI corresponds approximately to formula (18), whereas that recommended by Bjerrum (1972) and Pilot (1972) corresponds to formula (14) and (14a). Formula (14) leads to a stronger reduction for clays with $w_L = 50-90\%$ than the SGI reduction, whereas Fig. 22 indicates that the risk of failure is greater in clays with $w_L > 90\%$, even after a shear strength reduction according to SGI.

Customary factors of safety are associated with investigation and calculation methods used in engineering practice today and the introduction of new reduction factors may necessitate some alterations of the F_s-values to be used in geotechnical design. Determination of the relationship between the factor of safety and the probability of failure (eqn 32) makes it possible to calculate the optimum value of the factor of safety and in this way to find the best economic solution also when new reduction factors are used.

3 INTRODUCTION

The reliability of stability calculations based upon undrained shear strength values measured in soft clays has been a matter of concern since the introduction of the fall-cone test by the Geotechnical Commission of the Swedish State Railways. The strength values obtained in fall-cone tests were originally calibrated by the Secretary of the Geotechnical Commission, John Olsson, against the undrained shear strength calculated from loading tests on piles in soft clay, observations from failures of railway embankments and slopes, and results from unconfined compression tests. Direct correlations between strength values from fall-cone tests and the undrained shear strength measured in shear box tests and punching tests were established by Skaven-Haug (1931) in Norway and by Hultin (1937) and Caldenius (1938) in Sweden. The difference between the relationships found in these tests was explained as a result of different grain size distribution and organic content of the clay. Skaven-Haug´s tests on relatively coarse-grained Norwegian clays gave higher shear strength values than the Swedish tests on more fine-grained and organic clays (Caldenius 1938, Jakobson 1946).

The Swedish Geotechnical Institute adopted in 1946 a calibration formula for calculating the undrained shear strength according to fall-cone tests based upon mean values from the Norwegian and Swedish tests and recommended that the shear strength of organic clay should be reduced by 20% and the shear strength of gyttja (ooze) by 40% (Jakobson 1946).

However, the development of new standard piston samples made it later necessary to correct the original calibration of the fall-cone test through the introduction of a general reduction coefficient of 0.90 in the old calibration formula (Svenska Geotekniska Föreningen 1962).

After introduction of the field vane test it was assumed the shear strength obtained from vane tests (τ_v) would not require any similar reduction (Cadling and Odenstad 1950). However, as the τ_v-values were generally close to the shear strength τ_k determined by the fall-cone test, it soon became clear, that if special reduction coefficients has to be applied to the τ_k-values, similar coefficients should be used for the τ_v-values, too.

In 1969 it was concluded, that the same reduction coefficients should be applied to both the vane test and the fall-cone test (Statens Geotekniska Institut 1970. The difference between vane test and fall-cone test results at greater depths is mainly caused by sampling disturbance and stress changes during the sampling operation).

In 1972 Bjerrum (1972, 1973) published his recommendation for reduction of the τ_v-values, based upon an analysis of case records from failures of embankments, footings, loading tests and cuts in clay all over the world. According to Bjerrum the τ_v-values should be reduced also in non-organic clay, where no reduction is made according to the Swedish recommendation (Andréasson 1974).

Experience from loading tests, embankments and cuts in soft organic, plastic and varved clays indicate that the recommendations for reduction of the τ_k- and τ_v-values may still require some modifications. Swedish Highway Authorities have, for instance, based upon experience with highway embankments on black organic clays and silts containing iron sulphide in northern Sweden recommended reduction coefficients down to or even smaller than 0.5 for such organic soils. Experience from failures during the construction of the Kimola and Saima canals in Finland also indicate that the reduction methods used so far may in some cases give results which are on the unsafe side.

A total stress analysis using undrained shear strength has, of course, its limitations, and an effective stress analysis using the effective shear strength parameters c (or a = c/tanϕ) and ϕ is in many cases more reliable, specially in problems involving long-term stability (Janbu 1970). Because of its simplicity the total stress analysis has, however, a very widespread use in practice and it is therefore important to investigate the errors incorporated in this method and to develop correction methods or reduction factors in order to eliminate these errors.

4 DIFFERENT METHODS FOR REDUCING UNDRAINED SHEAR STRENGTH

4.1 Reduction of τ_k according to Swedish State Railways

The Geotechnical Commission of the Swedish State Railways introduced a relative strength number H equal to the mass in gram of a 60^o steel cone penetrating 4 mm into a clay sample when dropped from the surface of the sample (Statens Järnvägars Geotekniska Kommission 1922). John Olsson, the Secretary of the Commission, calculated the undrained shear strength τ_k from the formula (Jakobson 1946).

$$\tau_k = \frac{H_3}{C} \tag{1}$$

where C is a strength transferring coefficient and the index 3 refers to undisturbed soil samples (for remoulded samples the index 1 was used). If τ_k is in kPa, C is about 4 for a non-organic clay. Olsson found, that the C-value had to be increased in organic clay and gyttja (ooze) and suggested that this reduction of τ_k should be made on the basis of the cone liquid limit w_F (which is approximately equal to the percussion liquid limit w_L) determined by the fall-cone test (Fig. 1, Lindskog 1976).

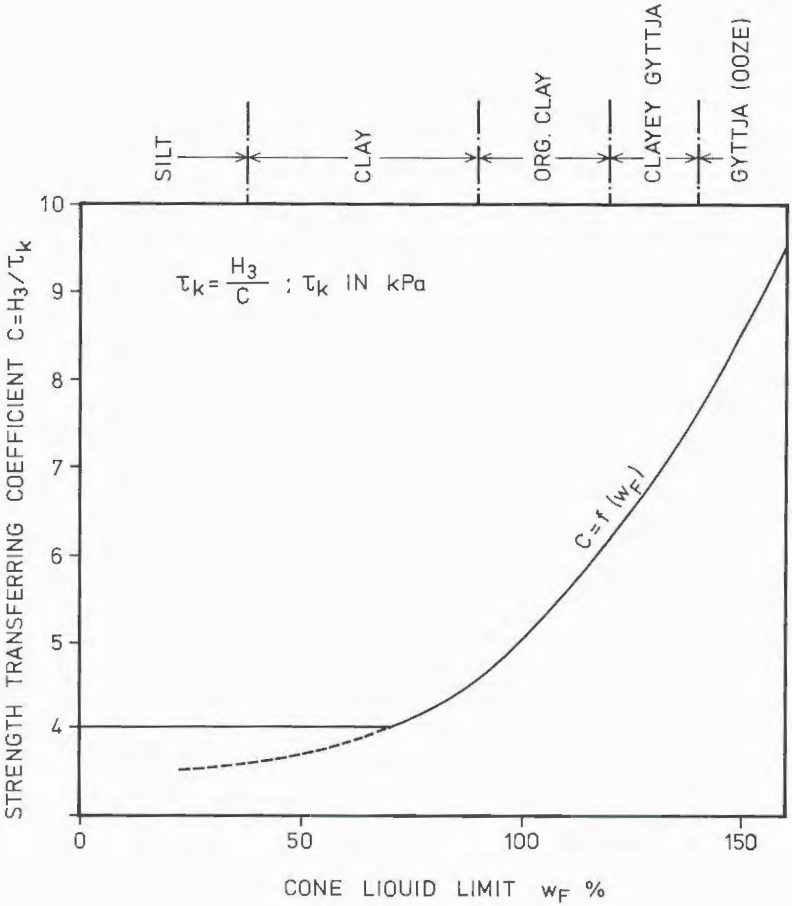


Fig 1 *Relationship between the cone liquid limit w_F and the strength transferring coefficient $C = H_3/\tau_k$ according to John Olsson when $H_3 < 150$ (H_3 = relative strength number of undisturbed clay obtained in fall-cone tests, τ_k = undrained shear strength from fall-cone test).*

Later on more detailed curves were worked out giving C as a function of both w_F and H_3 (see Fig. 5 in Osterman 1960). The strength number H_3 can be considered to correspond to the ultimate bearing capacity of the undisturbed clay sample in rapid loading and the coefficient C thus includes rate effects in transferring strength measured in rapid loading to normal loading rates in the construction of e.g. railway embankments.

4.2 Reduction of τ_k and τ_v according to SGI

Taking into account the experience and test results of Olsson, Skaven-Haug, Hultin and Caldenius the Swedish Geotechnical Institute recommended in 1946 that the undrained shear strength τ_k (in kPa) from fall-cone tests should be calculated from the calibration formula (Jakobson 1946).

$$\tau_k = \frac{H_3}{3.6+0.0064\ H_3} \tag{2}$$

but that the denominator in this formula should not be less than 4.0. In addition to this the undrained shear strength (τ_{fu}) to be used in stability calculations should be reduced depending on the organic content of the clay. According to the recommendations the reduction factor μ in the formula (3)

$$\tau_{fu} = \mu \cdot \tau_k \tag{3}$$

should be μ = 0.80 in organic clay and μ = 0.60 in gyttja (ooze).

Later on Hansbo (1957) deduced a new calibration formula for the cone test:

$$\tau_k = K \cdot \frac{Q}{h^2} \tag{4}$$

where Q is the mass of the fall-cone (in gram), h is the cone penetration (mm) and K is a cone factor depending on the angle of the cone and on the type of soil sampler. For the piston sampler SGI IV and the 100 gram 30° cone the cone factor is K = 10, when τ_k is in kPa.

The introduction of new soil samplers made it necessary to recalibrate the fall-cone test and this was done using the field vane test. In this connection it was found that both tests were of a similar character and that an interpretation from one test to another did not necessitate taking into account the organic content (Hansbo 1957, Osterman 1960). If a special reduction factor (μ in equation 3) was used for the fall-cone test, a similar reduction factor thus had to be applied to the vane test. SGI recommended in 1969 that the reduction factor is selected on the basis of the cone liquid limit w_F as shown in Table 1.

TABLE 1

Reduction factors for the undrained shear strength measured in fall-cone tests or field vane tests according to recommendations by SGI (Broms 1972).

Cone liquid limit w_F %	Reduction factor μ
80-100	0.90
100-120	0.80
120-150	0.70
150-180	0.60
>180	0.50

4.3 Reduction of τ_k and τ_v according to Swedish Road Authorities

Based on experience from failures of highway embankments on organic clays and other clays with high liquid limit the Swedish Road Authorities in 1968 deduced a formula for determination of the reduction factor μ on the basis of the cone liquid limit w_F. According to an investigation by Hansbo (1957) there exists for many Swedish clays an empirical relationship between the ratio τ_v/σ_o' and the cone liquid limit

w_F (= w_F %/100)

$$\tau_v/\sigma_o' = 0.45\ w_F \tag{5}$$

where τ_v is the undrained shear strength measured in field vane tests and σ_o' is the effective overburden pressure in the same depth. However, according to the Swedish Road Authorities, the undrained shear strength τ_{fu} to be used in stability calculations should not be greater than a certain limiting value (Olofsson 1976):

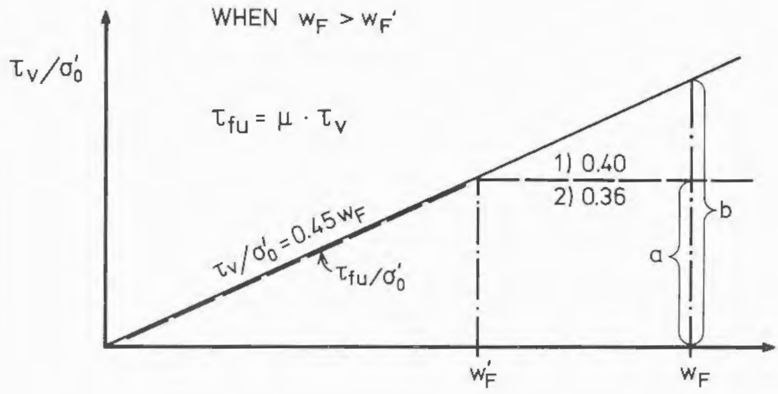
$$\tau_{fu} \leq 0.40\ \sigma_o' \tag{6}$$

This maximum value has been used in highly plastic clay from Mexico City and it agrees fairly well with the reduced τ_v/σ_o'-ratio according to Osterman (1960).

As shown in Fig. 2 the upper limit $\tau_{fu} = 0.40\ \sigma_o'$ would give a reduction factor $\mu = 0.89/w_F$. This value is reduced by the Swedish Road Authorities to (Statens Vägverk 1974):

$$\mu = \frac{0.80}{w_F} \tag{7}$$

which corresponds to $\tau_{fu} = 0.36\ \sigma_o'$, when the cone liquid limit is $w_F \geq 0.80$ (80%).

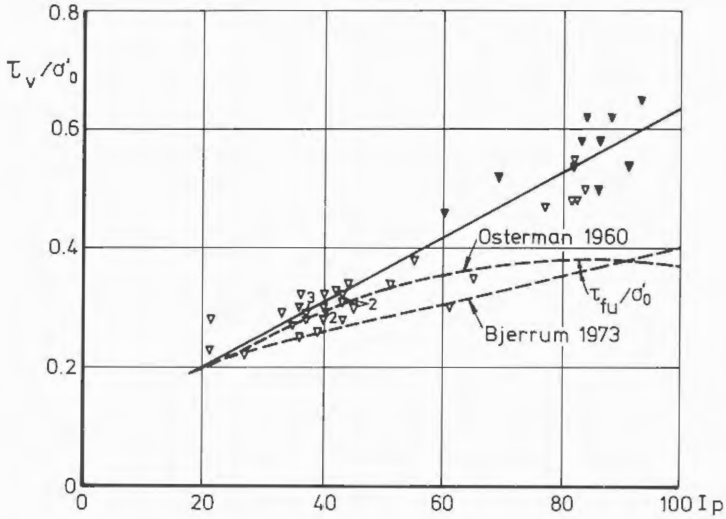


$$1) \quad \mu = \frac{a}{b} = \frac{0.40}{0.45\, w_F} = \frac{0.89}{w_F} \; ; \quad w_F' = 89\,\%$$

$$2) \quad \mu = \frac{0.36}{0.45\, w_F} = \frac{0.80}{w_F} \; ; \; w_F' = 80\,\% \; (a = 0.36)$$

Fig 2 Derivation of the reduction factor $\mu = 0,80/w_F$ recommended by Swedish Road Authorities.

Fig. 3 illustrates variations of the τ_v/σ_o'-ratio as a function of the plasticity index according to Osterman (1960). The reduced τ_{fu}/σ_o'-curve suggested by Osterman (reduced in a way similar to the reduction used by Olsson) and a curve corresponding to the reduction factor recommended by Bjerrum are also shown in Fig. 3.



τ_v = SHEARING RESISTANCE, VANE TEST

σ'_0 = OVERBURDEN PRESSURE, NORMALLY CONSOLIDATED

I_p = PLASTICITY INDEX

▽2 ▽3 = NUMBERS OF EQUAL TEST RESULTS

▽ = CLAY

▼ = MUDDY CLAY

▼ = HIGHLY MUDDY CLAY

Fig 3 *Ratio between shear strength (τ_v) from field vane tests and the effective overburden pressure (σ'_o) as a function of the plasticity index (I_p) according to Osterman (1960). The reduced ratio according to Osterman and Bjerrum is also shown in the figure.*

4.4 Reduction of τ_k according to Bjerrum

Bjerrum (1972, 1973) deduced his relationship between the reduction factor and the plasticity index I_p from an analysis of a number of case records from failures in soft clay in different parts of the world. In Fig. 4 these case records are plotted in a different way in order to obtain the factor of safety as a function of the liquid limit (Andréasson 1974). Some of the case records included in Bjerrum´s paper are omitted from Fig. 4, because of lack of information about the liquid limit or because of other uncertainties. The small circles in the figure represent embankment failures, the triangles are failures under footings and loading plates (∇) or strutted excavations (Δ) and the small squares represent failures of cuts and unsupported excavations.

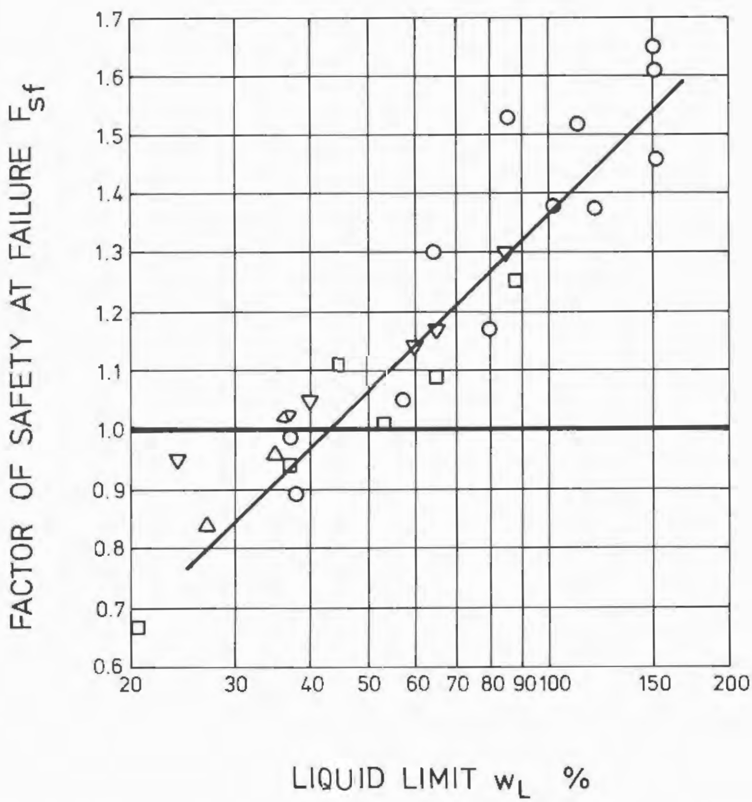


Fig 4 *Theoretical factor of safety at failure (F_{sf}) as a function of the liquid limit of the clay according to case records in Bjerrums General Report to the Moscow Conference in 1973. The undrained shear strength has been measured by field vane tests (Andréasson 1974).*

On the basis of the streight line in Fig. 4 the curve in Fig. 5 has been drawn, giving the reduction factor μ as a function of the liquid limit of the clay. The reduction factor recommended by SGI (Table 1) is also shown in Fig. 5. As seen from the figure the differences between the reduction factors recommended by Bjerrum and SGI are important for clays with low liquid limit, but for clays with high liquid limit the difference is rather small.

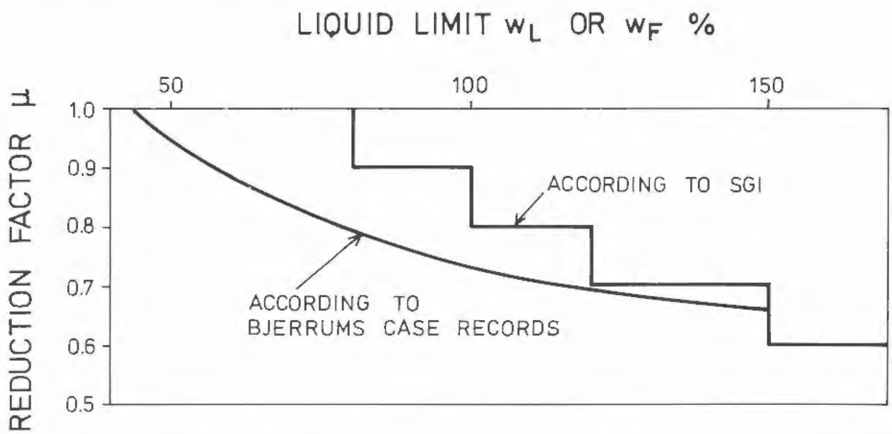


Fig 5 The reduction factor as a function of the liquid limit according to the case records in Fig 4 and according to SGI (Andréasson 1974).

4.5 Other reduction methods

In a report on embankment failures in France Pilot (1972) analysed a number of case records and stated as a conclusion, that the theoretical factor of safety at failure increases both with the liquid limit and with increasing plasticity index of the soil. The empirical equations for the factor of safety at failure F_{sf} deduced by Pilot (1972) are:

$$F_{sf} = 0.6\, w_L + 0.7 \qquad (8)$$

$$F_{sf} = 0.7\, I_p + 0.9 \qquad (9)$$

where w_L = liquid limit = w_L%/100 and I_p = plasticity index = I_p%/100.

Equations (8) and (9) can be used to calculate the reduction factor $\mu = 1/F_{sf}$, which should be applied to the shear strength values measured in field vane tests. Another possible procedure is to reduce the theoretical factor of safety by dividing it with the factor of safety at failure according to equation (8) or (9). The w_L- and I_p-values in these equations should then be calculated as mean values for the most dangerous slip surface.

A similar method has been applied by Aas (1976), who found a linear relationship between the factor of safety at failure F_{sf} and the ratio τ_v/σ_o'. This relationship can be expressed by the formula

$$F_{sf} = 2.7 \quad \tau_v/\sigma_o' + 0.38 \tag{10}$$

where τ_v is the mean (uncorrected) vane shear strength and σ_o' is the mean effective overburden pressure in the critical slip line. (In a homogeneous soil the τ_v/σ_o'-ratio can be determined at the center of gravity of the circular arc). The method recommended by Aas (1976) is simple and practical and gives results which are generally on the safe side. This method will be discussed later in Chapter 2.6 and 3.2.

A method for reduction of the shear strenght of overconsolidated clay samples has been worked out by Ladd and Foott (1974). The method is based on a non-linear relationship between the ratio τ_{fu}/σ_o' and the overconsolidation ratio σ_c'/σ_o'. This relationship is determined from laboratory tests on undisturbed soil samples in each special case. The method of Ladd and Foott (1974) is thus more laborous than previous methods, but it gives apparently more reliable results, as the extrapolation to other types of clay is avoided.

4.6 Comparison of different methods

The reduction factor proposed by Bjerrum (1972, 1973) can be expressed approximately by the formula

$$\mu_B = \frac{1.2}{1 + I_p} \tag{11}$$

where the plasticity index $I_p = I_p\%/100$.

The linear relationship found by Aas (1976) between the factor of safety at failure (F_{sf}) and the ratio τ_v/σ_o', equation (10), would on the other hand give the reduction factor

$$\mu_A = \frac{2.6}{1 + 7\,\tau_{v}/\sigma_o'} \tag{12}$$

If the reduction factor is expressed as a function of the liquid limit using the empirical relationships proposed for Swedish clays by Hansbo (1957): $\tau_v/\sigma_o' = 0.45\ w_L$ (equation 5) and Osterman (1960): $I_p = 0.82\ (w_L-0.20)$, the reduction factor μ_A and μ_B would be approximately:

$$\mu_A = \frac{2.5}{1 + 3\ w_L} \tag{13}$$

$$\mu_B = \frac{1.45}{1 + w_L} \tag{14}$$

For organic and plastic clays equation (13) gives considerably lower μ-values than equation (14), even if the last mentioned equation at high w_L-values gives slightly smaller reduction factors than according to the curve in Fig. 5.

If we calculate the relationship between the ratio τ_v/σ_o' and the liquid limit w_L, which corresponds to the case $\mu_A = \mu_B$, we find that this relationship differs from equation (5). If we take the starting point (μ = 1.0) of the reduction in both cases at $w_L = 0.5$ (this assumption is also in accordance with Pilot´s formula 8), which means that we take

$$\mu_B = \frac{1.5}{1 + w_L} \tag{14a}$$

the following approximate relationship is obtained

$$\tau_v/\sigma_o' = 0.10 + 0.25\ w_L \qquad (15)$$

The AB-line corresponding to equation (15) lies about half-way between the 0.45 w_L-line and the line representing the reduced shear strength τ_{fu} (Fig. 6). For clays with $\tau_v/\sigma_o' = 0.45\ w_L$ a greater reduction factor would thus have to be used, if the same τ_{fu}/σ_o'-curve is assumed to apply also to these clays:

$$\mu = \frac{2.5\ w_L + 1}{4.5\ w_L} \cdot \mu_B \qquad (16)$$

or

$$\mu = \frac{2.5\ w_L + 1}{3\ w_L\ (w_L+1)} \qquad (17)$$

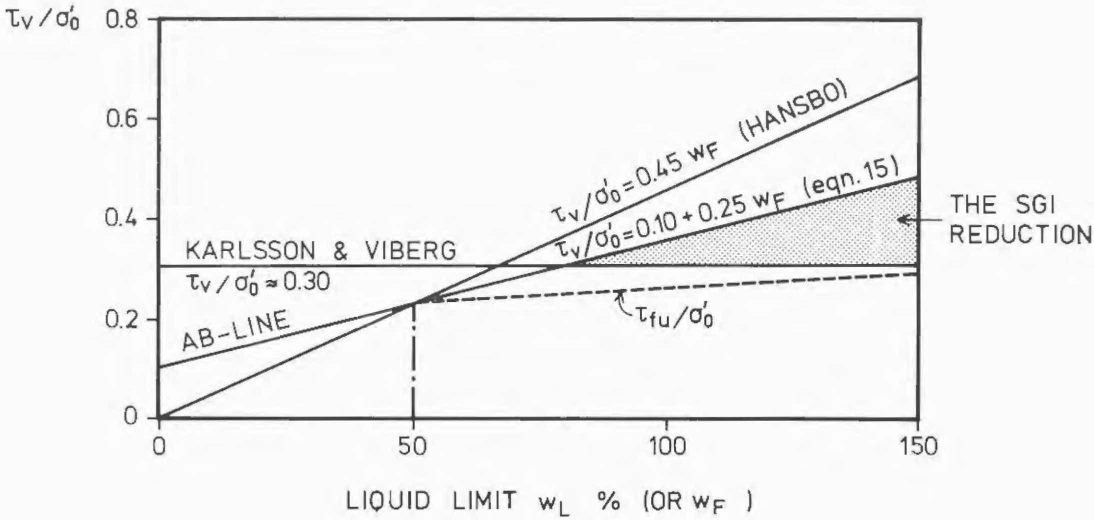


Fig 6 *Empirical and calculated relationships between the ratio τ_v/σ_o' and the liquid limit. The SGI-reduction corresponds to the area between the AB-line and the horizontal line $\tau_v/\sigma_o' = 0,30$.*

A comparison of Figs. 2 and 6 shows that reduction according to the τ_{fu}/σ'_o-curve in Fig. 6 would mean a considerably greater shear strength reduction than according to formula (7) and thus also a much greater reduction than according to SGI.

However, instead of using a statistically weak relationship between the τ_v/σ'_o-ratio and the liquid limit (or plasticity index), it seems preferably to use the actual τ_v/σ'_o-ratio directly as a basis for the reduction, according to formula (12) or by using the τ_v/σ'_o-scale in Fig. 7. The relation between the w_L-scale and the τ_v/σ'_o-scale in Fig. 7 is in accordance with equation (15). If the actual τ_v/σ'_o-ratio is smaller than according to equation (15) the reduction factor can be found using the w_L-scale in Fig. 7, if the actual τ_v/σ'_o-ratio is greater, the μ-value should be determined using the τ_v/σ'_o-scale.

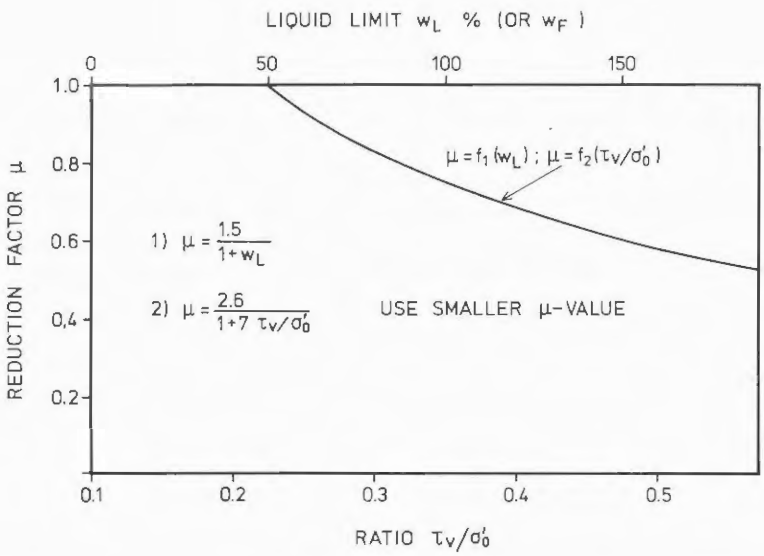


Fig 7 *Combined method for determination of the reduction factor μ. If $\tau_v/\sigma'_o \leq 0,10+0,25\ w_L$ the μ-value can be determined as a function of the liquid limit, if the τ_v/σ'_o-ratio is greater, the μ-value should be determined on the basis of the actual τ_v/σ'_o-ratio.*

The τ_{fu}/σ_o'-curve in Fig. 6 appears to asymptotically lean up with the horizontal line, where this ratio is equal to 0.30. The reduction factor corresponding to this line is

$$\mu = \frac{3}{1 + 2.5\ w_L} \tag{18}$$

This μ-value represents a reduction of the same magnitude as the SGI-reduction (Fig. 8). However, in order to avoid the scatter involved in the use of consistency limits it would be preferable also in this case to use the actual τ_v/σ_o'-ratio as a basis for the reduction instead of the (cone) liquid limit. Instead of equation (18) we then obtain the following simple formula

$$\mu = \frac{0.30}{\tau_v/\sigma_o'} \tag{19}$$

which gives the reduced shear strength $\tau_{fu} = 0.30\ \sigma_o'$.

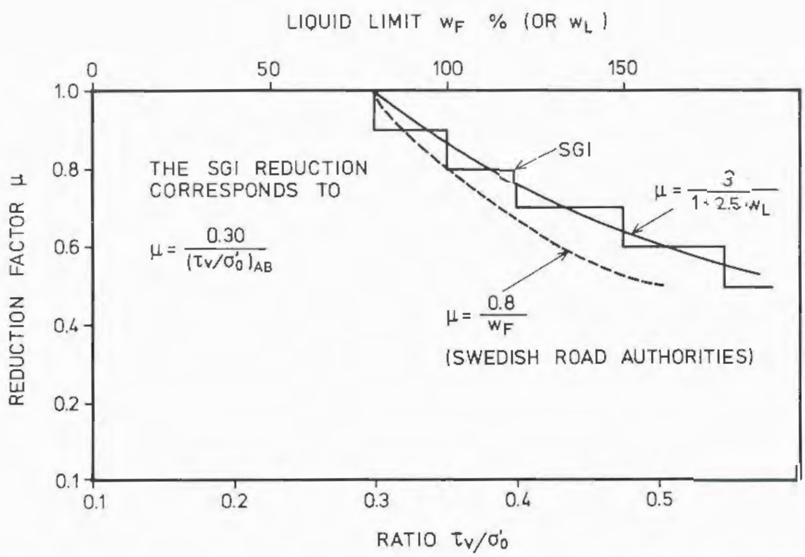


Fig 8 *Comparisons between the gradual reduction recommended by the Swedish Geotechnical Institute (SGI) and the μ-values according to equations (18) and (19). The reduction factor recommended by the Swedish Road Authorities is also shown in the figure.*

The "threshold value" 0.30 in formula (19) can apparently be different in different types of clay (according to Ladd and Foott it depends on the overconsolidation ratio and on the type of clay). A more correct value may be found by using the $\Delta\tau_v/\Delta\sigma_o'$-ratio corresponding to the $\tau_v = f(z)$ function in each special case. In this way we obtain the reduced shear strength

$$\tau_{fu} = \frac{\Delta\tau_v}{\Delta\sigma_o'} \cdot \sigma_o' \qquad (20)$$

where $\Delta\tau_v$ is the shear strength increase corresponding to a certain increase, $\Delta\sigma_o'$, of the effective overburden pressure σ_o'.

Fig. 9 shows a comparison of different reduction factors for an organic sulphide clay at Kalix in northern Sweden. It may be mentioned that the undrained shear strength according to plate loading tests at a depth of 2 to 4 m was 8-9 kPa (Schwab 1976) and that a full-scale test embankment gave the result $F_{sf} = 1.30$, when the (unreduced) shear strength from field vane tests was used (Holtz and Holm 1973).

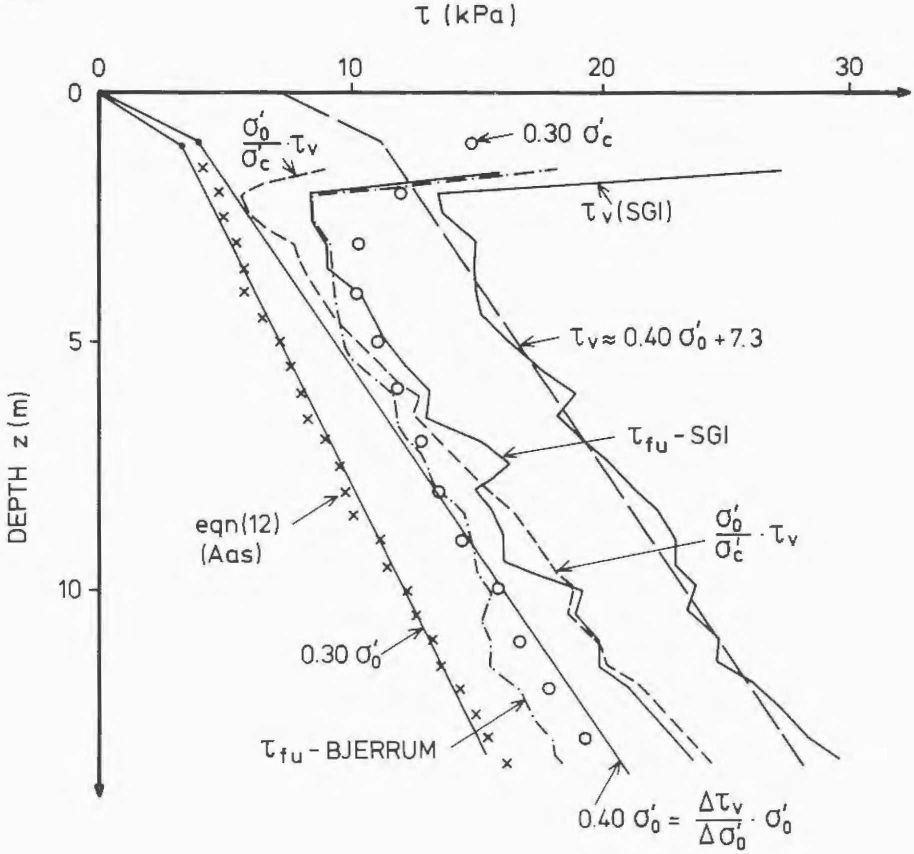


Fig 9 *Comparison between different reduction factors in organic sulphide clay at Kalix in northern Sweden.*

A comparison of the vane shear strength and the reduced shear strength ($\tau_v = 0.40\ \sigma_o' + 7.3$ kPa and $\tau_{fu} = 0.40\ \sigma_o'$, respectively, Fig. 9) shows, that reduction according to formula (20) means an elimination of the apparent cohesion 7.3 kPa. Changes of the effective overburden pressure during the period of construction, due to excavation, pore-water pressure induced by heavy rainfall etc. can be taken into account through the σ_o'-value. Application of formula (20) means then in principle use of an effective stress analysis with shear strength parameters estimated from field vane tests (from the $\tau_v(z)$-line).

If a step by step reduction is used, we obtain Table 2 instead of Table 1.

Table 2

Reduction factors according to formula (19), when the actucal τ_v/σ_o'-ratio is used as a basis for the reduction (instead of the liquid limit used in Table 1).

Ratio τ_v/σ_o'	Reduction factor μ
0.30-0.35	0.90
0.35-0.40	0.80
0.40-0.475	0.70
0.475-0.55	0.60
0.55-0.65	0.50
0.65-0.85	0.40
>0.85	0.30

Table 2 gives μ-values down to $\mu = 0.30$, corresponding to τ_v/σ_o'-ratios over 0.85. The step by step reduction is, however, rather crude, specially when $\mu < 0.50$. It is generally simpler to assume $\tau_{fu} = 0.30\ \sigma_o'$ or $\tau_{fu} = \Delta\tau_v/\Delta\sigma_o' \cdot \sigma_o'$.

It should be noted, that the curve for $\mu = 0.8/w_F$ in Fig. 8, used by the Swedish Road Authorities, does not correspond to the relationship $\mu = 0.36/(\tau_v/\sigma_o')$ (Fig. 2), but to the formula $\mu = 2/(10\tau_v/\sigma_o' - 1)$.

5 FACTORS AFFECTING SHEAR STRENGTH REDUCTION

5.1 Time effects

The influence of rate of strain on the shear strength measured in undrained tests has been discussed by many authors (see e.g. Bjerrum (1973), Hansbo (1972)). Fig. 10 illustrates the influence of time to failure (t_f) on the undrained shear strength (τ_{tf}) according to different authors. The curves representing Drammen and Lappo are from undrained triaxial tests, the lower curve from Skå-Edeby (Schwab 1976) shows the relationship between stress level and time to failure in screw-plate tests, while the other curves represent vane tests with varying rate of rotation (τ_o = shear strength at normal or standard rate).

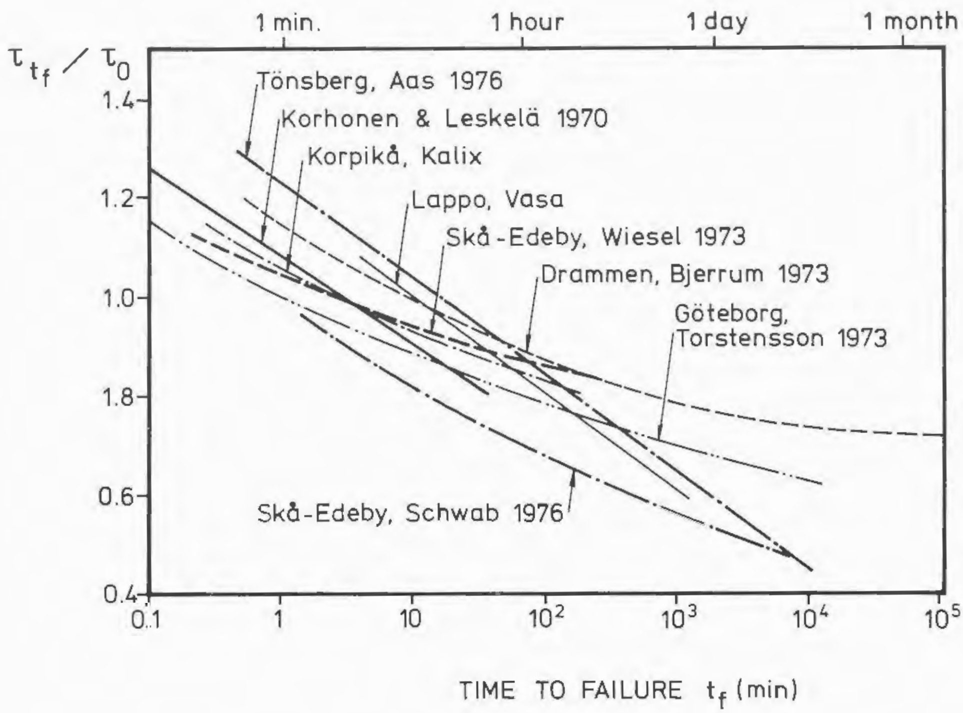


Fig 10 Relationship between stress level and time to failure in different clays.

As seen from Fig. 10 there is a considerable variation in the influence of time in different clays. In organic clays and clays of high plasticity the rate effects are generally greater than in lean and silty clays. However, the clay from Tönsberg, which shows a remarkably high rate effect (Aas 1976), has rather low plasticity (w_L = 37%, I_p = 13%). It is a slightly overconsolidated quick clay with high τ_v/σ_o'-ratio (0.4-0.7). High τ_v/σ_o'-ratios are usually found, except in highly sensitive clays, also in organic clays and sulphide clays (Hansbo 1957, Osterman 1960, Karlsson & Viberg 1967, Schwab 1976), which generally show great rate effects, too (the clays from Kalix and Lappo in Fig. 10 represent postglacial clays with organic sulphides, the test results of Korhonen and Leskelä (1970) represent mean values of postglacial Littorina clays).

Bjerrum (1973) has shown, that secondary or delayed consolidation under a period of hundreds or thousands of years will result in development of a quasi-preconsolidation pressure $\sigma_c' > \sigma_o'$. Also the τ_v/σ_o'-ratio will increase with time as a result of a more stable clay structure due to secondary consolidation. However, only a part of this increase of structural stability and strength can be utilized in stability calculations, since the positive effects of secondary consolidation will be partly destroyed by plastic deformation already before rupture (when the upper yield limit is passed (Keinonen 1963, Murayama & Shibata 1964, Foss 1968).

The τ_v-value corresponding to the preconsolidation pressure can thus be regarded as an apparent shear strength, which should be reduced by multiplying it with the reduction factor $\mu < 1.0$. The reduced shear strength τ_{fu} is generally equal to or greater than $\sigma_o' K_o \tan \phi'$, where σ_o' is the effective overburden pressure, K_o is the coefficient of earth pressure at rest and tan ϕ' is the coefficient of mobilized friction. In the same way the apparent shear strength (τ_v) can be regarded as a function of the preconsolidation pressure: $\tau_v = \sigma_c' K_o$ tan ϕ'. The reduction factor μ_T taking into account the influence of secondary consolidation should then have the lower limit

$$\mu_T \geq \sigma_o'/\sigma_c' \tag{21}$$

The quasi-preconsolidation pressure σ_c' resulting from secondary consolidation is generally less than 2 σ_o'. The influence of the overconsolidation ratio of overconsolidated clays can be taken into account according to the method proposed by Ladd and Foott (1974).

As the time effects are different in different soils, these effects should be determined, if possible, in each special case (at least in important projects), for instance by vane tests with varying rate of rotation. Another possibility is to measure the upper yield limit τ_y and take $\mu_T = \tau_y/\tau_v$ (Murayama & Shibata 1964, Korhonen & Leskelä 1970). It is, however, generally difficult to observe the upper yield limit in normal vane tests carried out with constant rate of rotation. Murayama and Shibata (1964) have proposed to measure the upper yield limit in controlled-stress tests on undisturbed samples in the laboratory. If the vane test has to be carried out as a controlled-stress test, a constant torque should be applied and the angle of rotation should be measured after certain time intervals.

According to Wiesel (1973) the time effects in field vane tests can be estimated on the basis of the modulus of shear deformation G. Determination of the modulus G would, however, require special laboratory tests or vane test equipment of the type used by Wiesel (1973).

5.2 Anisotropy

Vane tests with vanes of normal shape (height/diameter ration H/D = 2) are mainly influenced by the shear strength in vertical planes. In normally consolidated soils with normal stress anisotropy ($K_o < 1.0$) the shear strength in a horizontal plane is generally greater than in vertical planes, and the shear strenght τ_v measured in vane tests is then too small. Bjerrum (1973) has included the necessary anisotropy correction in his reduction factor. If we include possible effects of progressive failure, too, we may write

$$\mu = \mu_T \cdot \mu_A \cdot \mu_P \tag{22}$$

where μ_T is a factor correcting for the effects of time, μ_A is a factor correcting for anisotropy and μ_P is a factor correcting for progressive failure.

There are, however, cases where the shear strength is greater in vertical planes than in a horizontal plane and application of the combined reduction factor (see Fig. 9 in Bjerrum's (1973) report) may then lead to an error on the unsafe side. This was for instance the case at the Saima canal, where the shear strength measured in normal vane tests was 14% higher than the shear strength calculated from failures (Slunga 1973). As the anisotropy factor varies along the slip surface, depending on its inclination, and also depending on the direction of shear stress increment related to the shear stress before loading, it seems difficult to take the anisotropy effects into account simply by one single reduction factor. The influence of anisotropy should thus preferably be investigated in each special case, in the field by vane tests with vanes of different shape, and in the laboratory by direct shear tests and triaxial compression and extension tests on anisotropically consolidated undisturbed clay samples (Aas 1965, Berre & Bjerrum 1973, Ladd & Foott 1974).

If an empirical reduction factor is used, which is based on case records, this factor apparently also includes effects of anisotropy. Special attention to these effects should, however, be paid in case the soil has an unusual strength anisotropy, for instance if the shear strength in a horizontal plane is smaller than in vertical planes. The factor μ_A in equation (22) should then be determined by vane tests with vanes of different shape or other special tests. Bjerrum (1973) has estimated that the μ_A-factor included in his case records is μ_A = 1.0-1.2. If the shear strength in a horizontal plane is smaller than in vertical planes, this factor may be of the order of μ_A = 0.8-1.0.

Aas (1976) has compared the empirical relationship between the factor of safety at failure (F_{sf}) and the ratio τ_v/σ_o' from a number of case records (equation 10) with the relationship between the shear strength ratio τ_v/τ_L and τ_v/σ_o' (Fig. 11). The shear strength according to laboratory tests (τ_L) has been calculated as the mean value of reconsolidated undrained active (τ_A) and passive (τ_P) triaxial tests and direct shear tests (τ_D). In this way the effects of anisotropy are approximately incorporated in the τ_L-value (the τ_A-, τ_D- and τ_P-values are assumed to represent each one third of the critical slip surface). As seen from Fig. 11 the factor of safety at failure F_{sf} corresponds approximately to the shear strength ratio τ_v/τ_L, which means that the reduction factor is

$$\mu \approx \frac{\tau_L}{\tau_v} = \frac{\tau_A + \tau_D + \tau_P}{3\ \tau_v} \tag{23}$$

The τ_D-value obtained from direct shear tests with horizontal failure plane is, according to Aas (1976), approximately equal to the mean value from active and passive triaxial tests, which means that

$$\mu \approx \frac{\tau_A + \tau_P}{2\ \tau_v} \tag{24}$$

(This would apparently give as a third approximation $\mu \approx \tau_D/\tau_V$). Aas method for reduction of the shear strength seems thus primarily to be based upon anisotropy effects, whereas Bjerrum's method is based mainly on time effects. However, the time effects are actually incorporated in the equation (23) and (24) in such a way, that the time to failure in the laboratory shear tests is generally 1 to 3 hours, whereas the τ_V-value corresponds to a time to failure of only one or a few minutes.

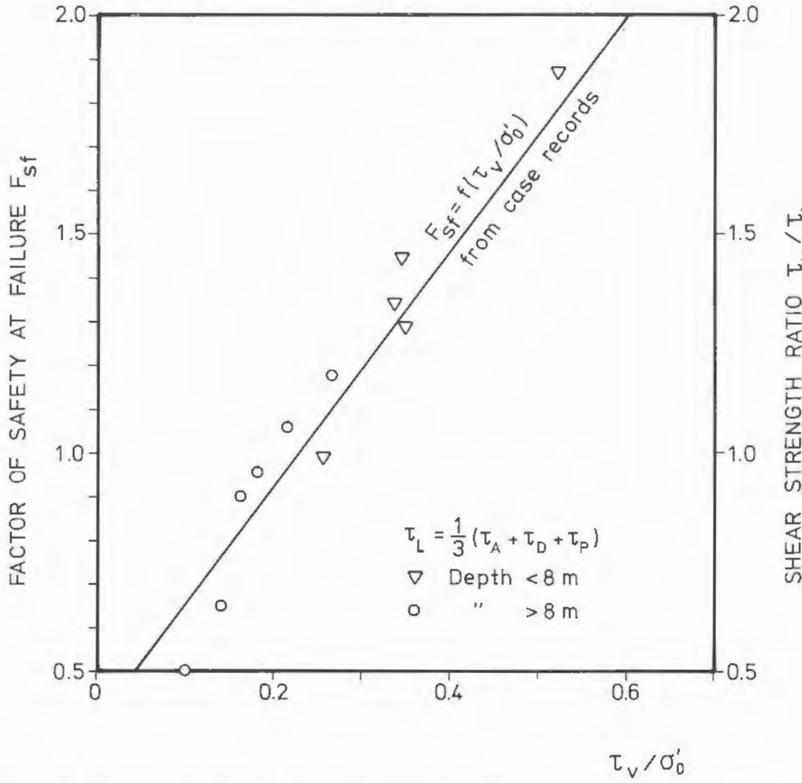


Fig 11 Ratio of shear strength from field vane tests (τ_v) and laboratory undrained shear tests (τ_L) compared with the linear relationship between the factor of safety at failure F_{sf} and the ratio τ_v/σ_o' according to Aas (1976). τ_L is calculated as mean value from active and passive triaxial tests (τ_A, τ_P) and direct shear tests (τ_D) on undisturbed samples reconsolidated to the stress state in-situ.

5.3 Progressive failure

If the variations of the undrained shear strength in a soil layer are great, the highest strength values are often ignored and the τ_{fu}-value to be used in a stability analysis is calculated, not as an arithmetical mean value, but as a reduced mean value, or even as a "mean minimum value" (Peck 1967, Zlatarev et al 1967). The reason for this procedure is generally the assumption that a failure may start at points where the shear strength is lowest compared to the shear stress and then proceeds as a progressive failure to areas with higher shear strength. The risk of progressive failure should be specially great in quick clay and other soils with high sensitivity and low residual strength.

From an analysis of the failure of a test embankment on sensitive clay in Canada the authors (Dascal et al 1972) concluded, that the theoretical factor of safety should be divided by a correction factor of 1.1-1.3 taking into account the possibility of a progressive failure. It is also suggested, that the correction factor should be related to the liquidity index of the clay. On the basis of results from a test fill on very soft quick clay at Ellingsrud in Norway Bjerrum (1973), on the other hand, concluded that progressive failure is a factor of minor importance and he did not propose any special correction factor taking into account the risk of progressive failure.

Progressive failure may in some cases be a factor of importance (Bishop 1967). Even if local overstressing does not necessarily result in a general shear failure, it may lead to local shear failure and greater displacements than what can be tolerated. This risk can be eliminated or reduced by using a "characteristic shear strength", which can be regarded as representative for the actual soil mass taking into account the amount and reliability of the geotechnical investigations and the stress-strain properties of the soil in each special case. One possible procedure for selection of the characteristic strength, used in concrete technology, is to take the mean value minus 145% of the standard deviation according to the shear strength measurements (Friis 1976). This would correspond to a reduction factor of

$$\mu = 1 - 1.45\ V_s \qquad (25)$$

where V_s is the coefficient of variation of the undrained shear strength. The V_s-value of cohesive soils may be of the order of 0.1-0.2 (Lumb 1970), which means that the reduction factor incorporated in the characteristic shear strength would be $\mu = 0.71-0.86$ (the corresponding F_s-value is about 1.15-1.40). More generally we may write

$$\mu = 1 - k_p \cdot V_s \qquad (26)$$

where k_p is a coefficient depending on the desired probability of failure.

Investigations by Vesic (1975) and Larsson (1976) indicate that the risk of progressive and local failure depends on the compressibility and stress-strain properties of the soil. This observation is supported by the fact that the risk of local failure is specially high in very compressible soils like peat (Helenelund 1975).

The influence of the residual shear strength (τ_r) may be analysed by using the brittleness index $I_b = (\tau_{fv} - \tau_r)/\tau_{fv}$ (Bishop 1967), where τ_{fv} is the peak value according to field vane tests, and the residual factor $R = (\tau_{fv} - \tau_{fu})/(\tau_{fv} - \tau_r)$ (Skempton 1964), where τ_{fu} is the actual undrained shear strength. The corresponding reduction factor for progressive failure μ_p is

$$\mu_P = 1 - R(1 - \tau_r/\tau_{fv}) \tag{27}$$

or $$\mu_P = 1 - R \cdot I_b \tag{28}$$

If there is no risk of progressive failure, we have $R = 0$ and $\mu_P = 1$, whereas for the case $R = 1$ (actual shear strength equal to the residual shear strength) we get $\mu_P = \tau_r/\tau_{fv}$. For the intermediate case $R = 0.5$ we have

$$\mu_P = (\tau_r + \tau_{fv})/(2\tau_{fv}) \text{ etc.}$$

5.4 Sampling disturbance

The undrained shear strength measured on clay samples in the laboratory is usually too small, because of sampling disturbance. Scandinavian piston samplers used before introduction of the standard piston sampler caused a disturbance corresponding approximately to a reduction factor of $\mu = 0.90$ (Chapter 1). Because of changes in the state of stress the strength reduction due to sampling disturbance increases generally with increasing depth. The shear strength measured for instance in fall-cone tests can therefore not be equal to the strength measured in field vane tests, if the depth is not taken into account in the calibration of the fall-cone test (Kallstenius 1963). According to Ladd and Lambe (1963) the mean negative pore pressure measured in "undisturbed" clay samples was about 20% smaller than the pressure corresponding to the stress state in-situ.

The field vane test is less influenced by disturbance effects. Pore pressure measurements indicate, however, that vane penetration causes important displacements and stress changes in the soil around the vane. The shear strength measured 1 day after installation of vane can be more than 20% higher than the shear strength obtained from ordinary tests, carried out about 5 minutes after installation of vane (Torstensson 1973).

The effects of soil structure disturbance during sampling and transportation of samples to the laboratory are generally more important in silty clay than in plastic clay. Sampling disturbance can also be important in organic clay containing plant fibres. The effective shear strength parameters are generally less affected by sampling disturbance than the undrained shear strength. The influence of sampling disturbance on the total stress analysis can be partly eliminated by the procedure recommended by Ladd and Foott (1974).

6 PROBABILITY OF FAILURE

6.1 Factors affecting risk of failure

There are many factors involving uncertainties in stability calculations, which all have to be covered by an adequate safety margin. Except uncertainties regarding the real shear strength of the soil and its variations in horizontal and vertical direction, time effects, anisotropy and risk of progressive or local failure, there may be uncertainties concerning climatic influences, pore-water pressures during wet seasons, variations of the ground water level, load variations depending on the method of construction, type of equipment, vibrations, assumptions regarding the unit weight, tolerances during excavation and filling, approximations in the stability analysis etc. In the following special attention will be paid to the influence of variations of the undrained shear strength and assumptions regarding the reduction factor μ.

The probability of failure along a certain slip surface can be expressed by the equation

$$P \quad (M_p - M_a) \leq 0 \tag{29}$$

where M_p is the resisting moment (mean value $\bar{M}_p$) and M_a is the overturning moment (mean value $\bar{M}_a$), which are assumed to be independent variables with normal distributions. The difference $X = M_p - M_a$ has thus also normal distribution; its mean value is $m = \bar{M}_p - \bar{M}_a$ and its standard deviation is $\sigma_x^2 = \sigma_{M_p}^2 + \sigma_{M_a}^2$, where σ_{M_p} and σ_{M_a} are the standard deviation of the resisting and overturning moments, respectively (Slunga 1972, Høeg & Murarka 1974).

The conditions under which the statistical factor of safety $\bar{F}_p \leq 1$ has the probability p% can be found by putting

$$\bar{M}_p - \bar{M}_a = \lambda_p \cdot \sigma_x \tag{30}$$

from which we get

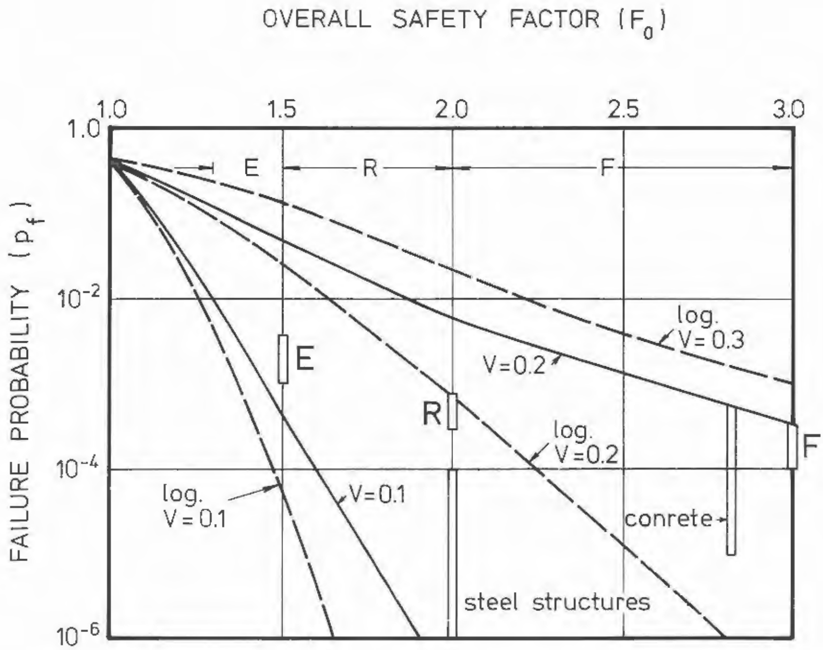
$$\frac{\bar{M}_p}{\bar{M}_a} - 1 = \frac{\lambda_p \quad \sigma_x}{\bar{M}_a} \tag{31}$$

or
$$\bar{F}_p = 1 + \lambda_p \frac{\sqrt{\sigma_{M_p}^2 + \sigma_{M_a}^2}}{\bar{M}_a} \tag{32}$$

The coefficient λ_p corresponding to the desired risk of failure (p%) can be found from standardized tables for variables with normal density distribution. It is thus possible to calculate the factor of safety $F_s = \bar{F}_p$ when failure occurs ($F_s \leq 1$) at a given probability p% by calculating the standard deviations σ_{M_p} and σ_{M_a}. If the number of observations (number of field vane tests or tests on undisturbed soil samples) is small, the coefficient λ_p in equation (32) can be replaced by the coefficient t_p, which can be found from tables for variables with t-distribution. The t_p-value depends on the number of observations and on the desired risk of failure.

Because of the variations and uncertainties of the undrained shear strength the standard deviation σ_{M_p} of the resisting moment is generally much greater than σ_{M_a}. The coefficient of variation $V_{M_p} = \sigma_{M_p}/\bar{M}_p$ may be of the order of 0.1-0.2 or smaller. As the coefficient of variation $V_{M_a} = \sigma_{M_a}/\bar{M}_a$ is smaller than V_{M_p}, also $\sigma_x/\bar{M}_a = \sqrt{\sigma^2_{M_p} + \sigma^2_{M_a}}/\bar{M}_a$ in equations (31) and (32) is generally of the order of 0.1-0.2.

Fig. 12 shows a comparison of the safety factor and the probability of stability failure according to Meyerhof (1970) for normal and log-normal density distributions and for coefficients of variation between 0.1 and 0.3. For earthworks with F_s = 1.3-1.5, normal distribution and V = 0.1-0.2 the probability of failure varies about 10^{-3} - 10^{-1} (p = 0.1-10%). As the risk of failure for instance for earthdams is greater than the risk which can reasonably be tolerated, Meyerhof (1970) suggested that the minimum overall safety factor should be increased to about 1.7, especially in connection with cohesive soils, unless careful performance observations are made both during and after construction.



E = EARTHWORKS

F = FOUNDATIONS

R = EARTH RETAINING STRUCTURES

V = COEFFICIENT OF VARIATION

Fig 12 Comparison between safety factor and probability of stability failure according to Meyerhof (1970).

6.2 Risk of failure according to case records

The probability of failure which can be expected if no reduction of the undrained shear strength is made is illustrated in Fig. 13, on the basis of the case records in Bjerrum's (1973) General Report to the Moscow Conference. According to Fig. 13 a factor of safety of 1.5 would correspond to a risk of failure of about 10%, which is equal to the upper limit of the probability of failure in earthworks mentioned in

Chapter 4.1. The risk of failure is considerably smaller, if the shear strength measured in field vane tests is reduced according to SGI and even more, if the reduction is made according to Bjerrum.

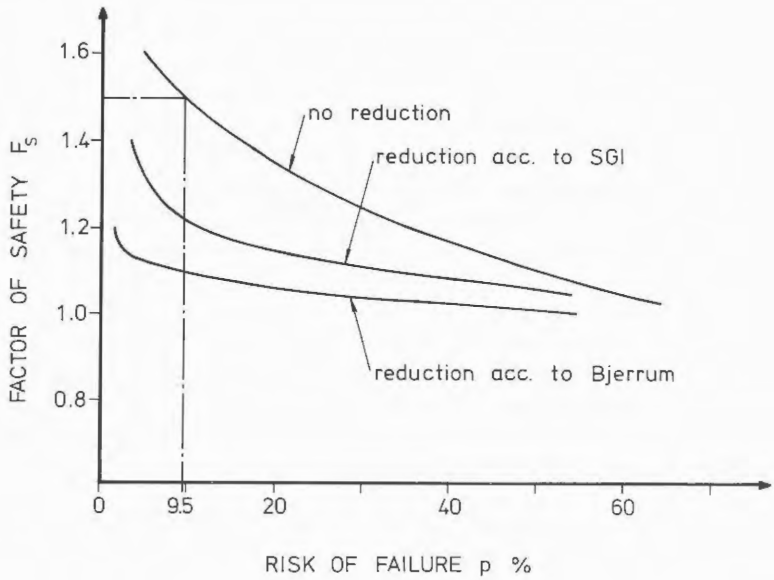


Fig 13 Risk of failure according to Bjerrums case records, when no reduction of the vane shear strength is made, respectively when τ_v *is reduced according to the method used by the Swedish Geotechnical Institute and the method proposed by Bjerrum.*

There are, however, a number of case records from failures showing greater variations (Pilot 1972, Ladd and Foott 1974). This is illustrated in Fig. 14, according to which specially case records from clays with high τ_v/σ_o'-ratio (e.g. organic and overconsolidated clays: 1 = Kalix $\tau_v/\sigma_o' = 0.53$, 3 = Kimola and 4 = Bara $\tau_v/\sigma_o' = 0.47$) show greater variations. The variations appear to be smaller, if the theoretical factor of safety at failure F_{sf} is expressed as a function of the τ_v/σ_o'-ratio (Fig. 15).

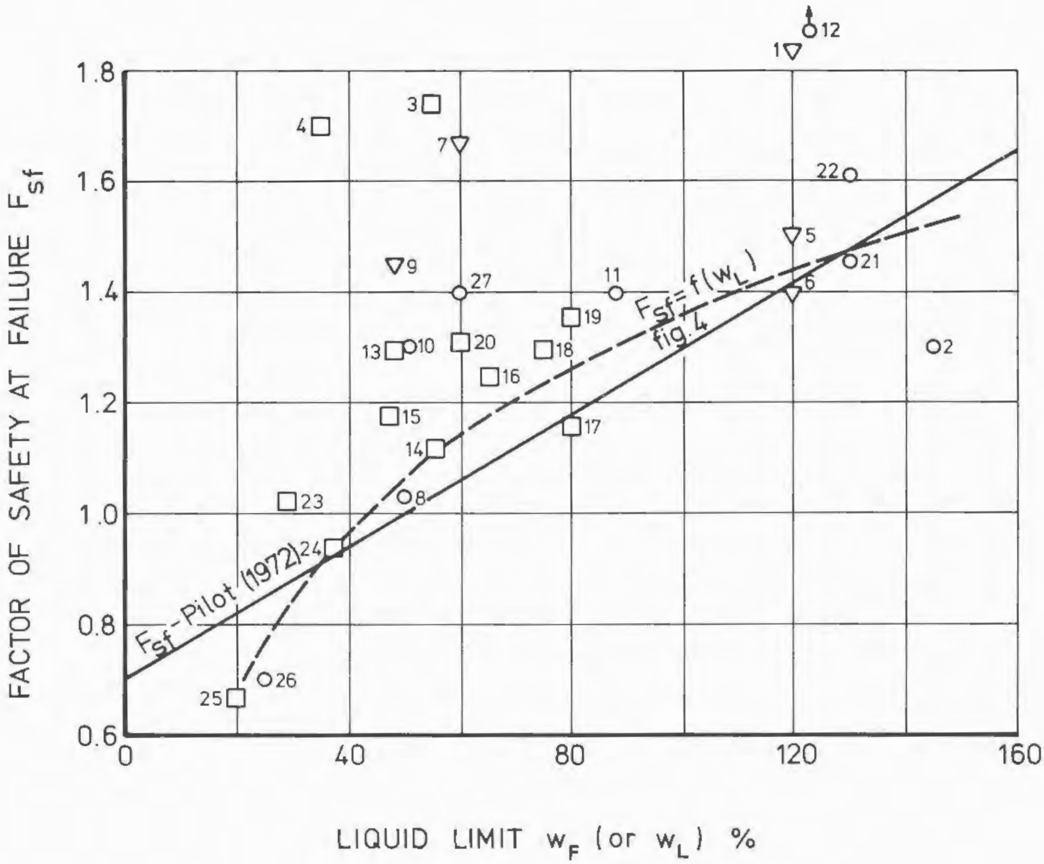


Fig 14 Relationship between the factor of safety at failure and the liquid limit of the clay according to Table 3. A considerable scatter can be observed, specially for case records representing clays with high τ_v/σ_o'*-ratio.*

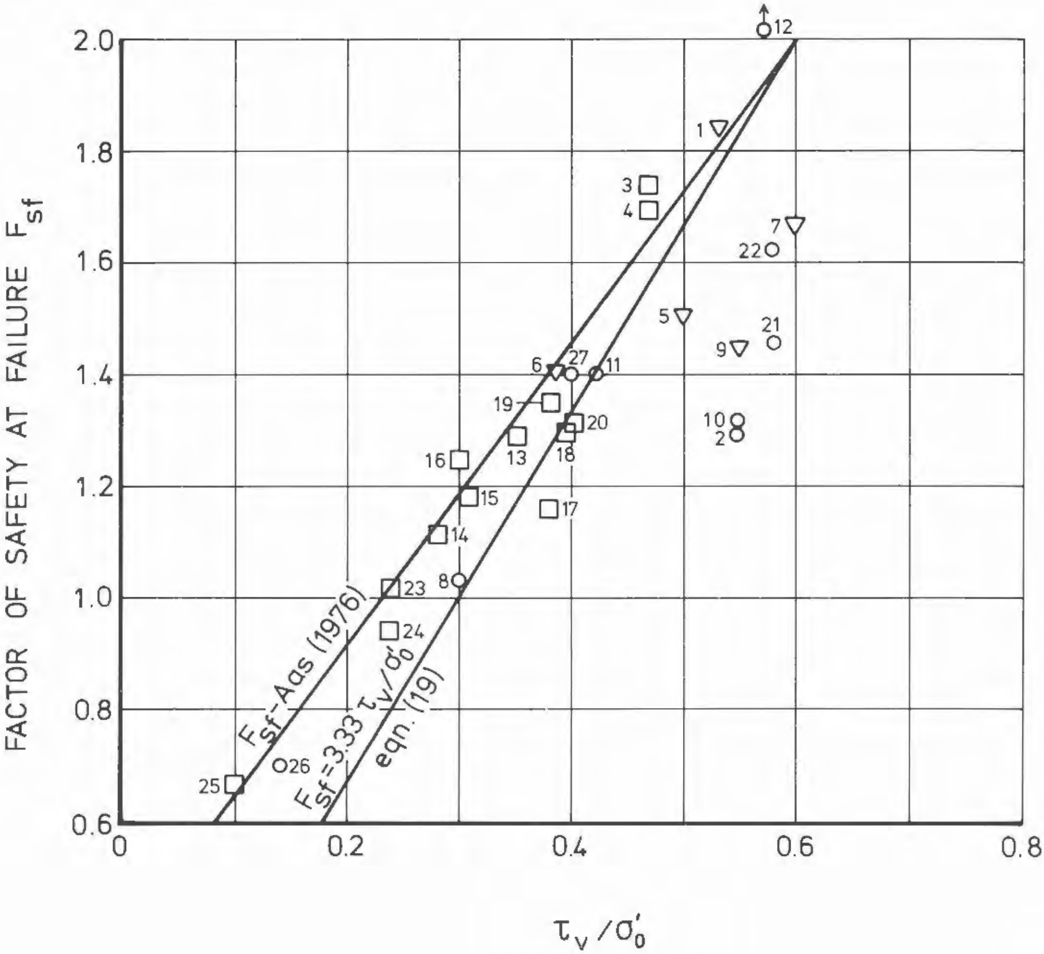


Fig 15 Relationship between the factor of safety at failure and the τ_v/σ_o'-ratio. The linear relationship according to formula (19) and that proposed by Aas (1976) are also shown in the figure.

Table 3 gives a summary of data for the case records in Figs 14 and 15. Most of the case records in Fig. 15 seem to fit rather well to the empirical F_{sf}-line proposed by Aas (1976), whereas the line representing $F_{sf} = 1/\mu$ according to formula (19) lies a little below most of the case records. There are, however, some case records from embankment failures which lie much below both lines and which apparently would fit better to a F_{sf} (w_L) function of the type suggested by Andréasson (1974) (Fig. 4).

The μ-values recommended by Bjerrum and SGI may thus be too high in organic and plastic clays with high apparent τ_v/σ_o'-ratio. This conclusion is supported by observations by e.g. Kankare (1969), Ladd and Foott (1974), Aas (1976) and Schwab (1976).

Clays with high τ_v/σ_o'-ratio, resulting from e.g. secondary consolidation, may thus need a greater strength reduction than the normal one according to equations (14a) and (18). It is suggested, that the shear strength reduction in such clays is based on the τ_v/σ_o'-ratio, equation (12) or (19), or using the τ_v/σ_o'-scale in Figs 7 and 8. The shear strength of overconsolidated clays can be reduced on the basis of the overconsolidation ratio according to Ladd and Foott (1974).

6.3 Risk of failure according to variations of the τ_v/σ_o'-ratio

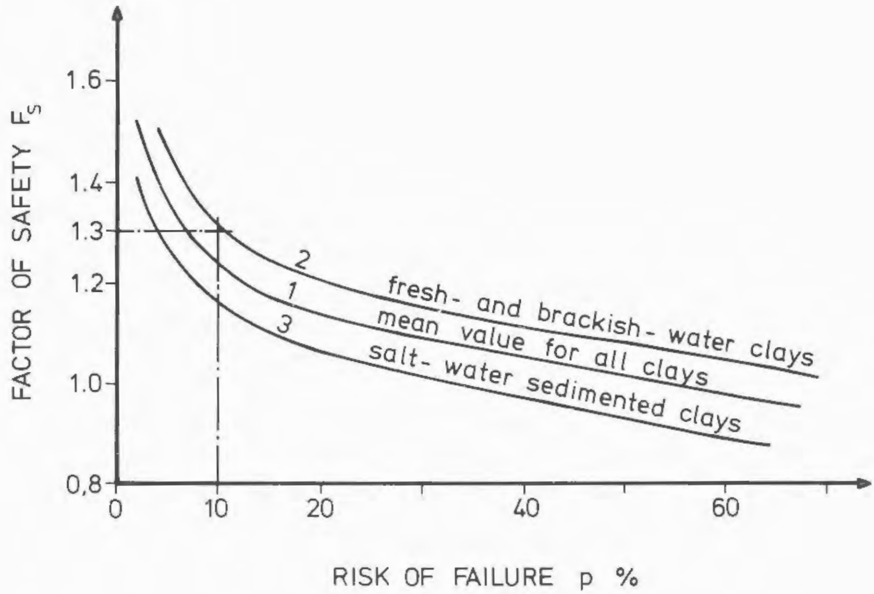
The reduction method proposed by the Swedish Road Authorities (Chapter 2.3) gives an interesting possibility to investigate the influence of variations of the relationship between the ratio τ_v/σ_o' and the consistency limits in different types of clays. Fig. 16 illustrates possible variations of the probability of failure in fresh- and brackish-water sedimented clays and salt-water sedimented clays according to an investigation of τ_v/σ_o'-variations in Swedish clays by

TABLE 3

Data for the case records in Figs 14 and 15 .

No	Site	$w_F(w_L)$ %	τ_v/σ'_o	F_{sf}	Remarks	References
1	Kalix	120	0.53	1.84	screw-plate	Schwab (1976)
2	"	145	0.55	1.30	embankment	Holtz & Holm (1973)
3	Kimola	55	0.47	1.74	canal slope	Kankare (1969)
4	Bara	35	0.47	1.70	excavation	Pusch (1968)
5	Skå-Edeby	120	0.50	1.50	test plate	Pusch, Hansbo (1972)
6	"	120	0.40	1.40	screw-plate	Schwab (1976)
7	Umeå	60	0.60	1.67	test plate	-"-
8	Kramfors	50	0.30	1.03	embankment	Jerbo & Sandegren (1962)
9	"	48	0.55	1.45	test plate	Schwab (1976)
10	"	50	0.55	1.30	embankment	-"-
11	Korpikå	88	0.42	1.40	embankment	Author (LuH)
12	"	122	0.57	2.20	slope & "	-"-
13	Göta R	48	0.35	1.29	river slope	Lindskog (SGI)
14	"	55	0.28	1.12	river slope	-"-
15	"	47	0.31	1.18	river slope	-"-
16	Göteborg	65	0.30	1.25	excavation	Alte (1976)
17	Mölndal	80	0.38	1.16	excavation	Andréasson (SGI)
18	Backa	75	0.39	1.30	excavation	Alte &Andréasson (1976)
19	Kållered	80	0.31	1.36	excavation	-"-
20	Saima	50	0.40	1.31	canal slope	Slunga (1972)
21	Bangkok	130	0.58	1.46	embankment	Eide & Holmberg (1972)
22	"	130	0.58	1.61	embankment	-"-
23	Mastemyr	29	0.24	1.02	excavation	Aas (1976)
24	Freia	37	0.24	0.94	excavation	-"-
25	Drammen	20	0.10	0.67	excavation	-"-
26	Ellingsrud	25	0.14	0.70	embankment	-"-
27	Onsöy	60	0.40	1.40	embankment	Berre (1976)

Karlsson and Viberg (1967). A clear difference seems to exist between different types of clays, resulting in a greater risk of failure in fresh- and brackish-water clays than in salt-water sedimented clays. This means that it would be motivated to use different reduction factors in different types of clays.



a/ FOR F_S = 1,30 : p_1 = 7 %; p_2 = 11 %; p_3 = 4 %

b/ FOR p = 10 % : F_{S1} = 1.24; F_{S2} = 1.32; F_{S3} = 1.16

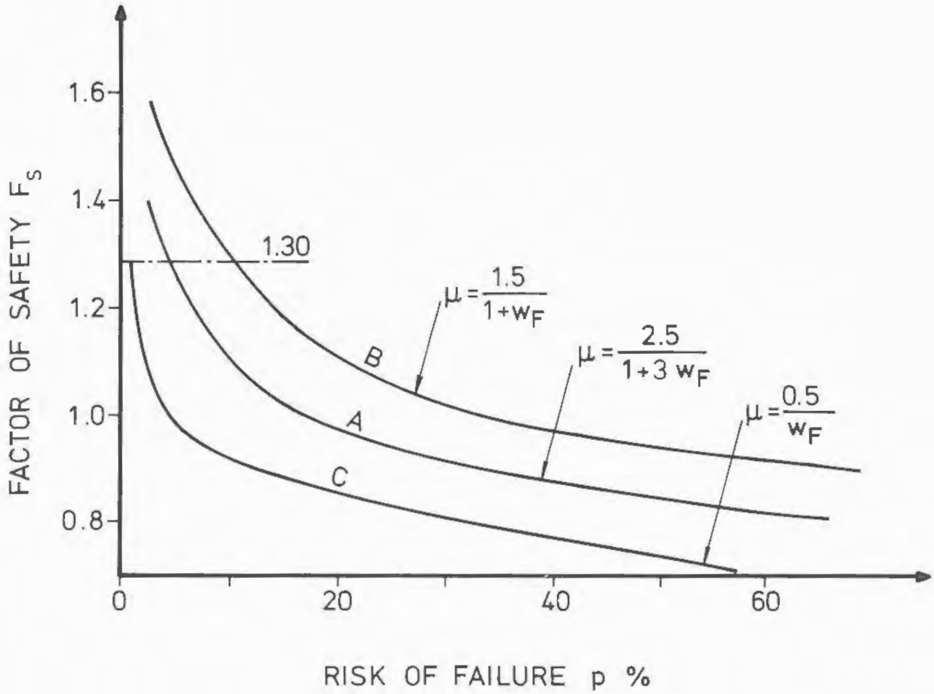
Fig 16 Possible variations of the risk of failure when the shear strength reduction is based upon one and the same empirical relationship between the ratio τ_v/σ'_o and the liquid limit in both fresh- and brackish-water clays and salt-water sedimented clays with w_F > 50% according to the statistical material of Karlsson and Viberg (1967).

It is interesting to note that the difference between the curve for fresh- and brackish-water clays and the curve for marine clays in Fig. 16 corresponds approximately to the difference between the two alternatives 1) and 2) in Fig. 2 and that the reduction factor proposed by the Swedish Road Authorities, equation (7), thus is more suitable for fresh- and brackish-water clays. However, the different starting points for the reduction (w_F' in Fig. 2) and the μ-value according to this method differ from those proposed by Bjerrum (1973), which should be more suitable for marine clays.

The difference between fresh- and brackish-water clays and salt-water sedimented clays observed by Karlsson and Viberg (1967) can be taken into account in equations (16) and (17) by using about 10% smaller μ-value for fresh- and brackish-water clays. The constant 3 in equation (17) should thus in these clays be replaced by 3.3.

Even if there is a clear difference between fresh- and salt-water sedimented clays in a locally limited area, this may not be the case globally, because of the great variations in geological and climatic conditions. The salt content of the pore-water may also decrease with time due to leaching, for instance in quick clays (Kenney et al 1967, Gardemeister 1975).

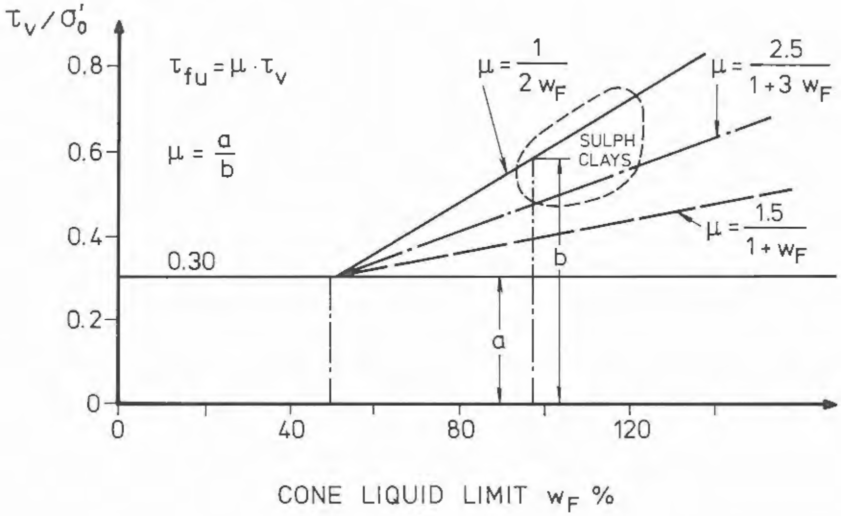
Fig. 17 illustrates variations of the probability of failure according to the same statistical material, when three different reduction factors are used. Curve A corresponds to a reduction factor according to formula (13) and curve B gives a reduction similar to formula (14a), whereas curve C means a stronger reduction, which would include even unfavourable cases like organic sulphide clays with high τ_v/σ_o'-ratio (Fig 18a). After a shear strength reduction according to curve B in Fig. 18b the risk of failure at a safety factor of $F_s = 1.30$ would still be rather high.



FOR $F_S = 1.30$; $p_A = 5\,\%$; $p_B = 11\,\%$; $p_C = 1\,\%$

Fig 17 Possible variation of the risk of failure in using different reduction factors based upon the cone liquid limit according to the same statistical material as used in Fig 16 ($w_F > 50\%$).

a/



b/

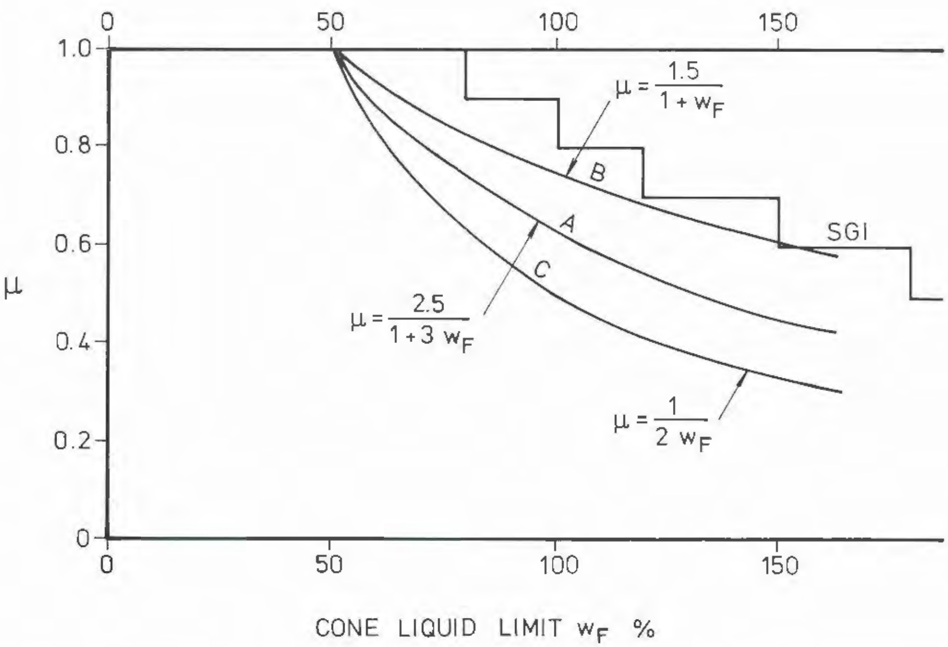


Fig 18 a) Derivation of approximate equations for the reduction factor for organic and plastic clays with different τ_v/σ_o'-ratio.
b) Comparison of different μ-formulas with the SGI-reduction.

There is a remarkable difference between the probabilities of failure according to Fig. 13 and Fig. 17. Fig. 17 indicates that there may exist cases with unfavourable conditions not included in the case records used by Bjerrum (1973), for instance organic clays with high apparent τ_v/σ_o'-ratio. Variations and uncertainties involved in the use of consistency limits may also introduce additional sources of error. If the most unfavourable cases including organic sulphide clays, which are of a more local interest (they occur mainly near to the Gulf of Botnia in northern Sweden and Finland, Schwab 1976) are excluded, the statistical risk of failure would be considerably smaller. This means that a reduction according to curve B in Fig. 18b would be sufficient in most cases and that a stronger reduction, for instance according to curve A, would be necessary mainly in local areas with more unfavourable soil conditions.

7 OPTIMUM VALUE OF THE FACTOR OF SAFETY

Determination of the relationship between the factor of safety and the probability of failure, equation (32), makes it possible to calculate the economic optimum value of the factor of safety (F_{opt}). This is illustrated in Fig. 19, showing the increase of construction costs (B) and the decrease of the risk of reconstruction (or repair-) costs (R), costs of damages on construction equipment (M) and possible human accidents (P), by increasing factor of safety. In this case the total costs have a minimum at a factor of safety of 1.50, which thus represents the optimum value F_{opt}. (The costs in Fig. 19 have been calculated from the construction costs of the Saima canal and the costs of repair of slope failures during the period of construction (Slunga 1973).

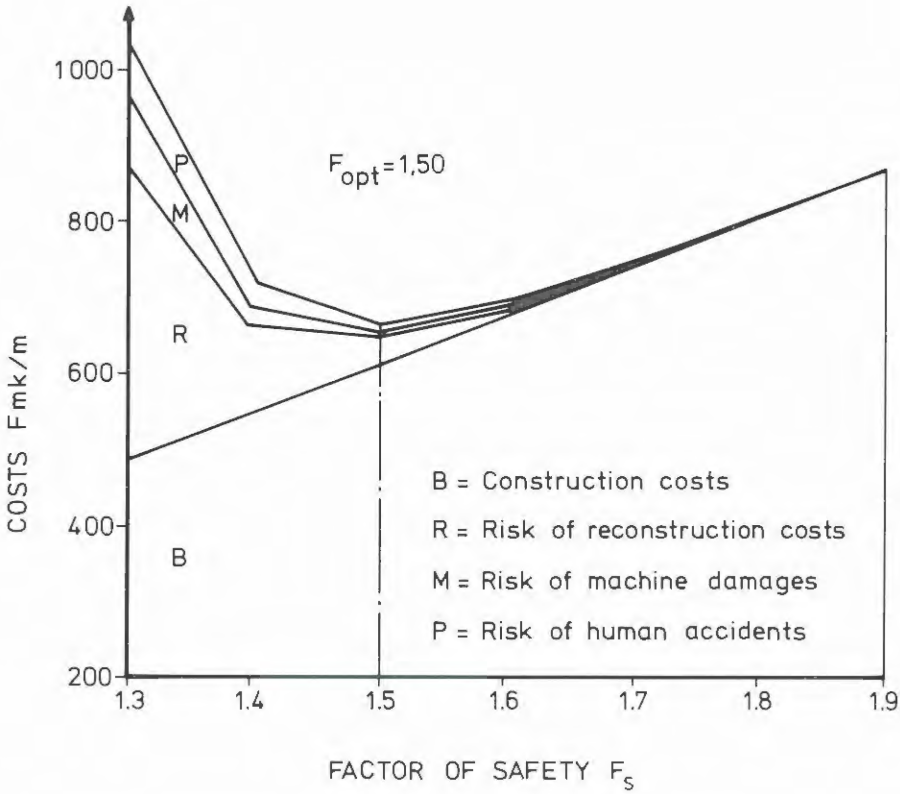


Fig 19 Determination of the optimum factor of safety F_{opt}. R-, M- and P-costs are proportional to the probability of failure.

The R-, M- and P-costs can vary within wide limits in different types of structures and also depend on during which period the failure occurs. A failure of a canal slope during the period of construction may only cause small repair costs, but a failure after this period can give rise to serious traffic disturbancies with high extra costs. A small slope failure during the construction of an earth dam may not be too serious, but the failure of an earth dam later on can cause disastrous floods and great losses of life and property. This means that the optimum factor of safety for long-term stability may be much higher than the F_{opt}-value applied in a short-term stability analysis.

Fig. 20 illustrates the increase of the optimum factor of safety by increasing costs ratio (R+M+P)/B. Structures like draining channels, which are easy to repair, may have a (R+M+P)/B-ratio of the order of 1.0 and a F_{opt}-value around 1.20 during the period of construction (short-term stability), whereas the long-term stability of an earth dam may correspond to a high costs ratio and the optimum value of the factor of safety may rise to 1.70 (Meyerhof 1970). The costs ratio may also include possible costs due to loss of prestige, reputation or public confidence of the client, consultant and/or contractor if a failure occurs (Kjellman 1940, Høeg & Murarka 1974).

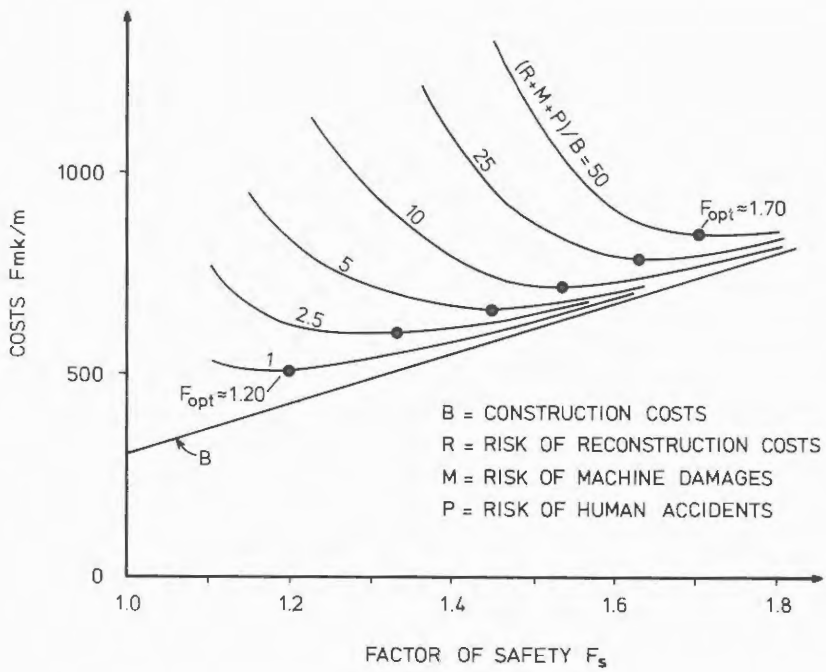


Fig 20 Influence of the costs ratio (R+M+P)/B on the optimum value of the factor of safety F_{opt} ($B = f(F_s)$ according to construction costs of the Saima canal).

Codes and standard specifications generally recommend minimum values of overall or partial factors of safety, which are based on methods of investigation and calculation normally applied in engineering practice (Hansen 1967). If these methods change, this also influences the factor of safety to be used in stability analysis. This fact is illustrated in Fig. 21, which shows economic optimum values of the factor of safety according to construction costs and costs of repair of a section of the Saima canal (at failure 2, Slunga 1972). When the total costs (B+R+M+P) are taken into account, an optimum value of F_{opt} = 1.40 is obtained, which lies within the limits normally recommended for earthworks (F_s = 1.30-1.50, Fig. 12). The anisotropy of the varved clay in this area is opposite to the normal stress anisotropy included in Bjerrums reduction factor μ, and if the shear strength is reduced taking both the time effect and the anisotropy into account, the optimum value will be F_{opt} = 1.15, which is lower than the factor of safety normally recommended.

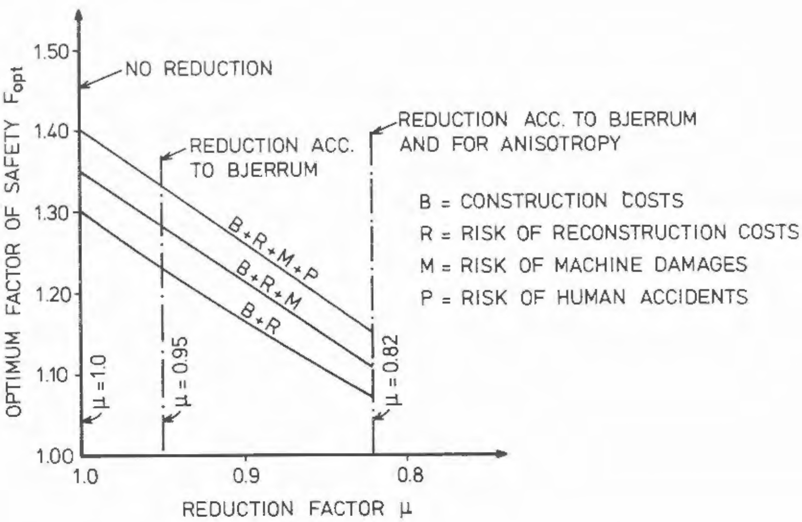


Fig 21 *Influence of shear strength reduction on the numerical value of* F_{opt} *during the period of construction of the Saima canal (short-term stability). Note the low values of* F_{opt} *after the* τ_v*-reduction.*

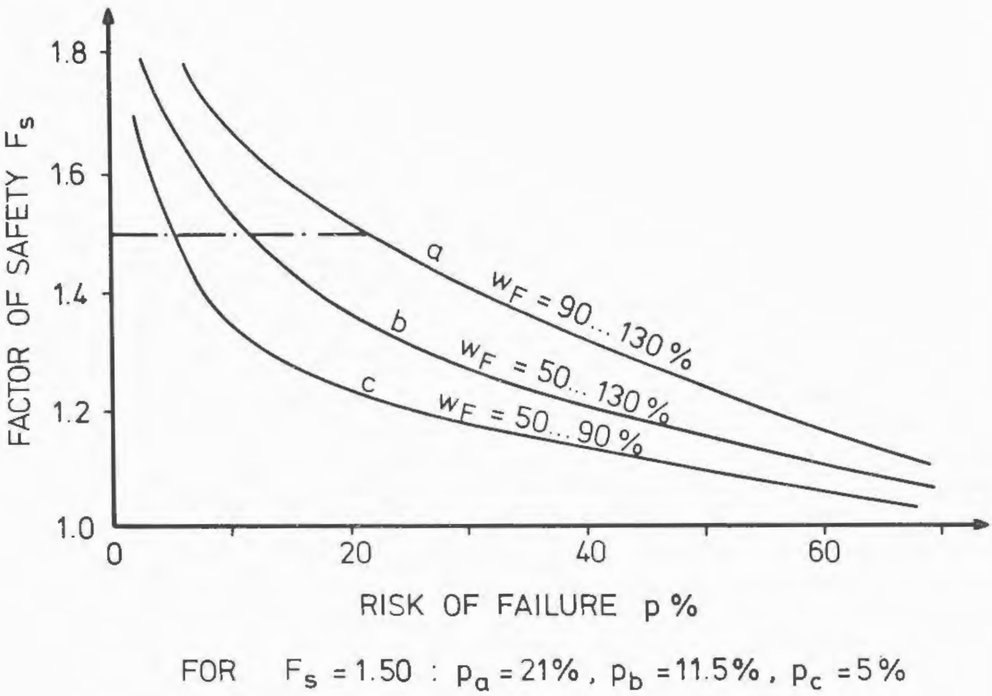


Fig 22 Risk of failure when the reduction is based upon the AB-line in Fig 6 according to the statistical material of Karlsson and Viberg in a) organic clays (w_F > 90%), b) all clays with w_F >50% and c) in non-organic clays (w_F = 50-90). The risk of failure is greater in organic clays.

8 REFERENCES

Aas G 1965, A study of the effect of vane shape and rate of strain on the measured values of in-situ shear strength of clays. Proc 6th Int Conf SMFE Vol 1 Montreal.

Aas G 1976, Totalspänningsanalyser, prinsipp, grunnlag. Norske Sivilingeniørers Forening. Kurs i Jordartsegenskaper - bestämmelse og anvendelse. Gol 20-22.5.1976.

Alte B & Andréasson L 1976, Private communication.

Andréasson L 1974, Förslag till ändrade reduktionsfaktorer vid reduktion av vingborrbestämd skjuvhållfasthet med ledning av flytgränsvärdet. CTH Intern report (not published).

Berre T 1976, Stabilitetsanalyse av prøvefylling i Onsøy. NIF-kurs Jordartsegenskaper Gol 20-22.5.1976.

Berre T & Bjerrum L 1973, Shear strength of normally consolidated clays. Proc 8th Int Conf SMFE Vol 1.1 Moscow.

Bishop A W 1967, Progressive failure - with special reference to the mechanism causing it. Proc Geot Conf Oslo 1967 Vol 2.

Bjerrum L 1972, Embankments on soft ground. Proc ASCE Spec Conf on Performance of Earth and Earth-Supported Structures Purdue Univ Vol 2.

Bjerrum L 1973, Problems of soil mechanics and construction on soft clays. State-of-the-art-report Session 4 Proc 8th Int Conf SMFE Vol 3 Moscow.

Broms B 1972, Jordarternas egenskaper. Kompendium i geoteknik. Statens Geotekniska Institut Medd 10 Stockholm.

Broms B 1973, Stability of embankments and excavations in soft clays, Discussion. Proc 8th Int Conf SMFE Vol 4.2 Moscow.

Cadling L & Odenstad S 1950, The vane borer. Swedish Geot Inst Proc 2 Stockholm.

Caldenius C 1938, Några rön från grundundersökningar i Göteborg rörande fasthetens variation inom lerorna. Tekn Tidskrift 1938:51.

Dascal O, Tournier J B, Tavenas F & LaRochelle P 1972, Failure of a test embankment on sensitive clay. Proc ASCE Spec Conf Purdue Univ Vol 1.

Eide O & Holmberg S 1972, Test fills to failure on soft Bangkok clay. Proc ASCE Spec Conf Purdue Univ Vol 1.

Foss I 1968, Tillsynelatende overkonsolideringseffekt og sekundaere setninger i normalkonsoliderte marine leirer. Väg- och vattenbyggaren 1968:8.

Friis J 1976, Innføring av grensetilstandsmetoder i geoteknikk. NIF-kurs Jordartsegenskaper Gol 20-22.5.1976.

Gardemeister R 1975, On engineering-geological properties of fine-grained sediments in Finland. Tech Res Centre Build Tech Publ 9 Helsinki.

Hansbo S 1957, A new approach to the determination of the shear strength of clay by the fall-cone test. Swedish Geot Inst Proc 14 Stockholm.

Hansbo S 1972, Bestämning av tillåten grundpåkänning med hänsyn till viskösa elasto-plastiska egenskaper hos lera. Nordisk Geoteknikermöte Trondheim.

Hansen J Brinch 1967, The philosophy of foundation design. Proc Symp Bearing Capacity and Settlement of Foundations Duke Univ.

Helenelund K V 1975, Geotechnical peat investigations. Proc 1st Baltic Conf SMFE Vol 1 Gdansk.

Holtz R D & Holm G 1973, Belastningsförsök på svartmocka. SGI Report (not published).

Hultin T 1937, Försök till bestämning av Göteborgslerans hållfasthet. Tekn Samfundets Handlingar 1937:2.

Höeg K & Murarka R P 1974, Probabilistic analysis and design of a retaining wall. Journ Geot Eng Div ASCE Vol 100 GT3.

Jakobson B 1946, Kortfattat kompendium i geoteknik 1946. Statens Geotekniska Institut Medd 1 Stockholm.

Janbu N 1970, Grunnlag i Geoteknikk. Tapir Forlag Trondheim.

Jerbo A & Sandegren E 1962, Skredet i Kyrkviken vid Kramfors 1-4 febr 1959. K Järnvägsst Geot avd Medd 7 Stockholm.

Kallstenius T 1963, Studies on clay samples taken with standard piston sampler. Swedish Geot Inst Proc 21 Stockholm.

Kankare E 1969, Geotechnical properties of the clays at the Kimola canal area with special reference to the slope stability. State Inst Tech Res Publ 152 Helsinki.

Karlsson R & Viberg L 1967, Ratio c/p' in relation to liquid limit and plasticity with special reference to Swedish clays. Proc Geot Conf Oslo 1967 Vol 1.

Keinonen L 1963, On the sensitivity of water-laid sediments in Finland and factors inducing sensitivity. State Inst Tech Res Publ 77 Helsinki.

Kenney T C, Moum J & Berre T 1967, An experimental study of bonds in a natural clay. Proc Geot Conf Oslo 1967 Vol 1.

Kjellman W 1940, Säkerhetsproblemet ur principiell och teoretisk synpunkt. IVA Handl 156 Stockholm.

Korhonen K-H & Leskelä A 1970, Siipikaira pohjatutkimusvälineenä (Vane apparatus as field investigation implement). State Inst Tech Res Report III:143 Helsinki.

Ladd C C & Lambe T W 1963, The strength of undisturbed clay determined from undrained tests. Spec Tech Publ 361 NRC-ASTM Symp Lab Shear Testing of Soils.

Ladd C C & Foott R 1974, New design procedure for stability of soft clays. Journ Geot Eng Div Proc ASCE Vol 100 GT 7.

Larsson R 1976, Skjuvhållfasthet och kritiska spänningar i lera. Statens Geot Inst Preliminär rapport Linköping.

Lindskog G 1976, Private communication.

Lumb P 1970, Safety factors and the probability distribution of soil strength. Canadian Geot Journ 1970:3.

Meyerhof G G 1970, Safety factors in soil mechanics. Canadian Geot Journ 1970:4.

Murayama S & Shibata T 1964, Flow and stress relaxtion of clays. Symp on Rheology and Soil Mechanics Grenoble.

Olofsson T 1976, Private communication.

Osterman J 1960, Views on the stability of clay slopes. Swedish Geot Inst Reprint and Prelim Reports 1 Stockholm.

Peck R B 1967, Bearing capacity and settlement, Certainties and uncertainties. Proc Symp Bearing Capacity and Settlement of Foundations Duke Univ.

Pilot G 1972, Study of five embankment failures on soft soils. Proc ASCE Spec Conf Purdue Univ Vol 1.

Pusch R 1968, Skred i överkonsoliderad lera. Väg- och vattenbyggaren 1968:8.

Schwab E 1976, Bearing capacity, strength and deformation behaviour of soft organic sulphide soils. R Swed Inst of Technology (KTH thesis) Stockholm.

Skaven-Haug 1931, Skjaerfasthetsforsøk med leire. Medd Norges Statsbaner 1931:6.

Skempton A W 1964, Long-term stability of clay slopes. 4th Rankine Lecture Geotechnique 1966:2.

Slunga E 1972, Om statistisk säkerhet hos jordslänt i anisotropisk lera. Nordisk Geoteknikermöte Trondheim.

Slunga E 1973, Discussion. Proc 8th Int Conf SMFE Vol 4.3 Moscow.

Statens Geotekniska Institut (SGI) 1970, Reducering av skjuvhållfasthet med avseende på finlekstal och sulfidhalt. Sammandrag av teknisk träff 1969-12-11 (not published).

Statens Järnvägars Geotekniska Kommission 1914-22 1922, Slutbetänkande. Statens Järnvägar Geot Medd 2 Stockholm.

Statens Vägverk 1974, Bankpålning. Särtryck ur verksamhetshandboken 1974-12 TV121 Stockholm.

Svenska Geotekniska Föreningens (SGF) Kolvborrkommittē 1962, Tolkning av fallkonprov på provkroppar av lera tagna med standardkolvborr (not published).

Torstensson B-A 1973, Kohesionspålar i lös lera. En fältstudie. Byggforskningen Rapport R38:1973 Stockholm.

Vesic A S 1975, Bearing capacity of shallow foundations (in Foundation Engineering Handbook, Ed H F Winterkorn, Van Nostrand) New York.

Wiesel C-E 1973, Some factors influencing in-situ vane test results. Proc 8th Int Conf SMFE Vol 1.2 Moscow.

Zlatarev K, Stefanoff G & Bejkoff M 1967, Experience from inadequate construction of a dike. Proc Geot Conf Oslo 1967 Vol 1.

STATENS GEOTEKNISKA INSTITUT
Swedish Geotechnical Institute
Fack
S-581 01 Linköping
Tel: 013/11 51 00

RAPPORT/REPORT

No.		år	Pris kr (Sw.crs.)
1.	Grundvattensänkning till följd av tunnelsprängning. *P. Ahlberg, T. Lundgren*	1977	50:-
2.	Påhängskrafter på långa betongpålar. *L. Bjerin*	1977	50:-
3.	Methods for reducing undrained shear strength of soft clay. *K.V. Helenelund*	1977	30:-

STATENS GEOTEKNISKA INSTITUT
Swedish Geotechnical Institute
Fack
S-581 01 Linköping
Tel: 013/11 51 00